Introduction

1. The not-fully-appreciated Minkowski

The major reason for the publication of Minkowski's papers on spacetime physics is to try to correct an injustice – Minkowski's contributions to modern physics have not been fully and appropriately appreciated. For example, not only the general public, but even students of physics appear to believe that the physics concept of spacetime was introduced by Einstein. What is worse is that experts themselves also sometimes use (in lectures, classes, papers and books) versions of the expression "Einstein's spacetime."[1] This is both unfortunate and unfair, especially given the fact that Einstein initially resisted the spacetime physics introduced by Minkowski (for more details see below). Also, the very fact that so far his papers, which laid the foundations of spacetime physics, have not been published together either in German or English (and even his *Das Relativitätsprinzip* and *A Derivation of the Fundamental Equations for the Electromagnetic Processes in Moving*

[1] Although I prefer not to give any references, here is a relatively recent example from a lecture on special relativity in the Perimeter Institute: "Learning to use Minkowskian geometry to understand, very simply, a variety of aspects of Einstein's spacetime" (https://www.perimeterinstitute.ca/outreach/students/virtual-issyp/virtual-issyp-modern-physics/modern-physics-special-relativity-0).

Bodies from the Standpoint of the Theory of Electrons have not been translated into English so far) is an indication of the lack of proper appreciation of Minkowski's contributions.

Since the first publication[2] in April 1908 of Minkowski's mathematical formalism of what he regarded as a theory of an absolute four-dimensional world there have been attempts to downplay his revolutionary contributions to spacetime physics. Here are several examples:

- Unfortunately, it was Einstein himself (with Jakob Laub) who expressed the first documented[3] reservation towards Minkowski's four-dimensional physics. Einstein and Laub indicated in the first paragraph of their first paper on Minkowski's study *Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern* that "In view of the fact that this study makes rather great demands on the reader in its mathematical aspects, we do not consider it superfluous to derive here these important equations in an elementary way, which, is, by the way, essentially in agreement with that of Minkowski"[4]. Einstein called Minkowski's approach "superfluous learnedness"[5]

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[2] H. Minkowski, Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern, *Nachrichten der K. Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse* (1908) S. 53-111. This is the lecture Minkowski gave at the meeting of the Göttingen Scientific Society on December 21, 1907.

[3] A. Einstein, J. Laub, Über die elektromagnetischen Grundgleichungen für bewegte Körper. *Annalen der Physik* **26** (1908) S. 532-540; Über die im elektromagnetischen Felde auf ruhende Körper ausgeübten ponderomotorischen Kräfte. *Annalen der Physik* **26** (1908) S. 541-550.

[4] *The Collected Papers of Albert Einstein, Volume 2: The Swiss Years: Writings, 1900-1909* (Princeton University Press, Princeton 1989), p. 329.

[5] A. Pais, *Subtle Is the Lord: The Science and the Life of Albert Einstein* (Oxford University Press, Oxford 2005) p. 152.

(überflüssige Gelehrsamkeit). At that time Einstein had apparently had difficulty realizing the depth of Minkowski's ideas which probably explains his initial reservation and even hostility towards Minkowski's four-dimensional physics. Sommerfeld's recollection of what Einstein said on one occasion appears to confirm Einstein's negative attitude towards Minkowski's results: "Since the mathematicians have invaded the relativity theory, I do not understand it myself any more."[6]

- Sommerfeld understood and accepted Einstein's special relativity thanks to Minkowski's four-dimensional formulation. That is why it is difficult to explain why he made changes to the original text of Minkowski's lecture *Das Relativitätsprinzip* given at the meeting of the Göttingen Mathematical Society on November 5, 1907, which he prepared for publication in 1915. Sommerfeld's changes were favourable to Einstein as Pyenson[7] observed: "Sommerfeld was unable to resist rewriting Minkowski's judgement of Einstein's formulation of the principle of relativity. He introduced a clause inappropriately praising Einstein for having used the Michelson experiment to demonstrate that the concept of absolute space did not express a property of phenomena. Sommerfeld also suppressed Minkowski's conclusion, where Einstein was portrayed

[6] A. Sommerfeld, To Albert Einstein's Seventieth Birthday. In: *Albert Einstein: Philosopher-Scientist.* P. A. Schilpp, ed., 3rd ed. (Open Court, Illinois 1969) pp. 99-105, p. 102.

[7] L. Pyenson, Hermann Minkowski and Einstein's Special Theory of Relativity, *Archive for History of Exact Sciences* 17 (1977) pp. 71-95, p. 82; see also L. Corry, Hermann Minkowski and the Postulate of Relativity, *Archive for History of Exact Sciences* **51** (1997) p. 273-314, p. 276 and P. L. Galison, Minkowski's Space-Time: From Visual Thinking to the Absolute World, *Historical Studies in the Physical Sciences*, **10** (1979) pp. 85-121, p. 93.

as the clarifier, but by no means as the principal expositor, of the principle of relativity." Giving credit to Einstein for realizing the crucial role of the Michelson experiment is especially unfortunate since Einstein himself stated the opposite: "In my own development, Michelson's result has not had a considerable influence. I even do not remember if I knew of it at all when I wrote my first paper on the subject (1905). The explanation is that I was, for general reasons, firmly convinced that there does not exist absolute motion and my problem was only how this could be reconciled with our knowledge of electrodynamics. One can therefore understand why in my personal struggle Michelson's experiment played no role, or at least no decisive role."[8] Minkowski's view of the role of Einstein's 1905 paper in clarifying the physical meaning of the Lorentz transformations is expressed at the end of the first

[8]A. Pais, *Subtle Is the Lord: The Science and the Life of Albert Einstein* (Oxford University Press, Oxford 2005) p. 172. It does appear that Einstein had been unaware of the Michelson-Morley experiment when one reads carefully what he wrote in his 1905 paper: "Examples of a similar kind, and the failure of attempts to detect a motion of the earth relative to the "light medium," lead to the conjecture that not only in mechanics, but in electrodynamics as well, the phenomena do not have any properties corresponding to the concept of absolute rest, but that in all coordinate systems in which the mechanical equations are valid, also the same electrodynamic and optical laws are valid, as has already been shown for quantities of the first order." [*The Collected Papers of Albert Einstein, Volume 2: The Swiss Years: Writings, 1900-1909* (Princeton University Press, Princeton 1989), p. 140.] EInstein does not seem to have in mind the Michelson-Morley experiment in the phrase "the failure of attempts to detect a motion of the earth relative to the "light medium"..." because he talks about a *conjecture*: "the conjecture that not only in mechanics, but in electrodynamics as well, the phenomena do not have any properties corresponding to the concept of absolute rest;" had he been aware of the Michelson-Morley experiment he would have indicated that attempts, involving electromagnetic phenomena, also failed the discover the absolute motion.

part of his 1908 paper *The Fundamental Equations for Electromagnetic Processes in Moving Bodies* (see this volume): "The paper of Einstein which has been cited in the Introduction, has succeeded to some extent in presenting the nature of the transformation from a physical standpoint."

- Despite his initial negative reaction towards Minkowski's four-dimensional physics Einstein relatively quickly realized that his revolutionary theory of gravity would be impossible without the revolutionary contributions of Minkowski. At the beginning of his 1916 paper on general relativity Einstein wrote: "The generalization of the theory of relativity has been facilitated considerably by Minkowski, a mathematician who was the first one to recognize the formal equivalence of space coordinates and the time coordinate, and utilized this in the construction of the theory." This quote is hardly from the new 1997 translation.[9] Quite strangely, the first page of the paper containing the recognition of Minkowski's work had been omitted in the first English translation.[10]

- Many physicists (including relativists) do not appear to have been fully appreciating the depth of Minkowski's four-dimensional physics and his general explanation of relativistic phenomena – "The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as

[9] *The Collected Papers of Albert Einstein, Volume 6: The Berlin Years: Writings, 1914-1917* (Princeton University Press, Princeton 1997), p. 146.

[10] H. A. Lorentz et al., *The Principle of Relativity*, translated by W. Perrett and G. B. Jeffery (Methuen 1923; Dover repr., 1952).

interrelations between these worldlines" (this volume). In 1960 Synge wrote: "It is to support Minkowski's way of looking at relativity that I find myself pursuing the hard path of the missionary. When, in a relativistic discussion, I try to make things clearer by a space-time diagram, the other participants look at it with polite detachment and, after a pause of embarrassment as if some childish indecency had been exhibited, resume the debate in their own terms"[11]. Now the situation does not appear to be that bad, but it is not much better either – everyone can check how many kinematical relativistic effects are explained through spacetime diagrams in recent textbooks on relativity. Given the fact that it is *only* Minkowski's four-dimensional physics that provides *the correct explanations* of the relativistic effects (see below and also the next section), it is difficult to understand the reluctance and sometimes even resistance against explaining the kinematical relativistic effects as manifestations of the four-dimensionality of the world as Minkowski advocated. A possible but disturbing explanation may be an approach that appears to be held by some physicists – that it is merely *a matter of description* whether we will use Einstein's or Minkowski's versions of special relativity. I think such an approach is a sure recipe for a double failure – in genuinely understanding the profound physical phenomena, studied by spacetime physics, and in making discoveries in physics – because it is certainly not a matter of description whether the world (at the macroscopic scale) is three- or four-dimensional.[12]

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[11] J. L. Synge, *Relativity: the general theory* (North-Holland, Amsterdam 1960) p. IX.

[12] The dimensionality of the world is one of its most fundamental features, which is regarded to be on equal footing with its very existence.

- There have been authors of books on general relativity, spacetime and gravitation, including of recent (21st century) ones, who abundantly use Minkowski's four-dimensional mathematical formalism and spacetime concepts introduced by him, but in a whole book mention his name just once, for example. Again, I prefer not to give any references.

- What is also unfortunate is that some well-known physicists who write papers and books for the general public virtually do not mention Minkowski's contributions and often omit even his name. As a result most who have read about spacetime appear to believe it was introduced by Einstein.

- There have been claims by different authors that Minkowski did not understand Einstein's special relativity. The actual situation had been just the opposite as will be shown in the next section.

2. Minkowski and Einstein

Let me make it clear right away – it is not my intention at all to try to downplay Einstein's contributions to special relativity. As stated at the beginning of the Introduction the main purpose of this book is to correct an injustice towards Minkowski, and an injustice cannot be corrected by committing another injustice. I hope it would be fair to both Minkowski and Einstein to shed some additional light (based on the historical facts we know now) on what they knew and understood in the period 1905-1908. I think the best approach in such situations is to imagine that they both were alive and would read what is written about them.

Let me start with very brief information about Minkowski's academic background (Einstein's background is well-known) and several facts.

In April 1883 the French Academy granted the Grand Prize in Mathematics jointly to the eighteen year old Hermann Minkowski for his innovative geometric approach to the theory of quadratic forms and to Henry Smith. Thirteen years later, in 1896, Minkowski published his major work in mathematics *The Geometry of Numbers*.[13]

By 1905 Minkowski was already internationally recognized as an exceptional mathematical talent. At that time he became interested in the electron theory and especially in an unresolved issue at the very core of fundamental physics – at the turn of the nineteenth and twentieth century Maxwell's electrodynamics had been interpreted to show that light is an electromagnetic wave, which propagates in a light carrying medium (the luminiferous ether), but its existence was put into question since Michelson's interference experiments failed to detect the Earth's motion in that medium. Minkowski's documented involvement with the electrodynamics of moving bodies began in the summer of 1905 when he and his colleague and friend David Hilbert co-directed a seminar in Göttingen on the electron theory. The paper of Minkowski's student – Einstein – on special relativity was not published at that time; *Annalen der Physik* received the paper on June 30, 1905. Poincaré's longer paper "Sur la dynamique de l'électron" was not published either; *Rendiconti del Circolo matematico di Palermo* received it on July 23, 1905 and it appeared in 1906. Also, "Lorentz's 1904 paper (with a form of the transformations now bearing his name) was not on the syllabus."[14]

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[13] H. Minkowski, *Geometrie der Zahlen* (Teubner, Leipzig 1896).

[14] S. Walter, Minkowski, Mathematicians, and the Mathematical Theory of Relativity, in H. Goenner, J. Renn, J. Ritter, T. Sauer (eds.), *The Expanding Worlds of General Relativity*, Einstein Studies, volume 7, (Birkhäuser, Basel 1999) pp. 45-86, p. 46.

Minkowski's student Max Born, who attended the seminar in 1905, recalled in 1959 what Minkowski had said during the seminar:[15] "I remember that Minkowski occasionally alluded to the fact that he was engaged with the Lorentz transformations, and that he was on the track of new interrelationships." Three years later (in 1962) he was more specific:[16] "Minkowski's first ideas about relativity were already worked out and shown" at the seminar. Again Born wrote in his autobiography about what he had heard from Minkowski after Minkowski's lecture "Space and Time" given on September 21, 1908:[17] "He told me later that it came to him as a great shock when Einstein published his paper in which the equivalence of the different local times of observers moving relative to each other were pronounced; for he had reached the same conclusions independently but did not publish them because he wished first to work out the mathematical structure in all its splendour. He never made a priority claim and always gave Einstein his full share in the great discovery."

Max Born's recollections appear to confirm what the results of Minkowski's publications strongly imply – that it is virtually certain that Minkowski arrived independently at what Einstein called special relativity and at the concept of spacetime,[18] but Einstein and Poincaré published first.

[15] Quoted from T. Damour, "What is missing from Minkowski's "Raum und Zeit" lecture," *Annalen der Physik* **17** No. 9-10 (2008), pp. 619-630, p. 626.

[16] Quoted from S. Walter, "Hermann Minkowski's approach to physics," *Math Semesterber* (2008) **55**, pp. 213-235, p. 222

[17] M. Born, *My Life: Recollections of a Nobel Laureate* (Scribner, New York 1978) p. 131.

[18] Again the facts: at the seminar in the summer of 1905, when Minkowski shared elements of his research in "four-dimensional physics" (as he put it in his 1908 lecture), neither Einstein's nor Poincaré's papers were published – *Annalen der Physik* received Einstein's paper on June 30, 1905, whereas *Rendiconti del Circolo matematico di Palermo* received Poincaré's paper on July 23, 1905.

The fully developed by Minkowski mathematical formalism of the four-dimensional spacetime physics (the formalism we now use) is the best proof of that. It is true that Minkowski reported his results in his December 1907 lecture *The Fundamental Equations for Electromagnetic Processes in Moving Bodies* and published them in 1908 as a 59-page treatise. But mathematicians and experts in spacetime physics know that such a mathematical aparatus could not have been created in just several months.

Minkowski's results demonstrate that in the period 1905-1908 he found a truly revolutionary resolution of the difficult issues surrounding the electrodynamics of moving bodies – that the relativity principle implies, as will be briefly summarized below, that the Universe is a four-dimensional world with time as the fourth dimension.

So in the fall of 1907 Minkowski was the only one who had genuine understanding of a number of difficult and unresolved at that time issues:

- The profound *physical meaning of the relativity principle* – that physical phenomena are the same for all inertial observers in relative motion. As a mathematician it may have been easier for Minkowski (than for Einstein) to postulate that the (real) time t of a stationary observer and the abstract mathematical time t', which Lorentz introduced calling it the *local time* of a moving observer, are equivalent and to explore the consequences of such a hypothesis. Unfortunately, we will never know how Minkowski arrived at the idea that t and t' should be treated equally. What appears certain is that his path had been different from Einstein's. The mathematical way of thinking surely had helped Minkowski to realize that if two observers in relative motion have different times they necessarily must have different spaces as well (since space is perpendicular to time), which is impossible in a three-dimensional world, but in a four-dimensional world with time as the fourth dimension. Here is how Minkowski in his own words at his lecture *Space and Time* explained how he had realized the profound *physical meaning of the relativity principle* – that the world is four-dimensional. In the case of two inertial reference frames in relative motion along their x-axes "one can call t' time, but then must necessarily, in connection with this, define space by the manifold of three parameters x', y, z in which the laws of physics would then have exactly the same expressions by means of x', y, z, t' as by means of x, y, z, t. Hereafter we would then have in the world no more *the* space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics" (this volume). Minkowski suddenly found the answers to many questions in his four-dimensional physics, e.g. the answer to the question of why the relativity principle requires that physical phenomena be the same in all inertial reference frames – this is so because every inertial observer describes the phenomena in *exactly the same way* – in his own reference frame (i.e. in terms of his own space and time) in which he is *at rest*. Also, the answer to the question of the failure of Michelson's experiments to detect the motion of the Earth appears obvious – the Earth is at rest with respect to its space and therefore not only Michelson's but any other experiments would confirm this state of rest. As every observer always measures the velocity of light (and anything else) in his own (rest) space and by using his own time, the velocity of light is the same for all observers.

- Minkowski's realization that the relativity principle implies many times and spaces, which in turn implies that the world is four-dimensional, naturally explained why there is no absolute motion (since there are many spaces, not just one absolute space), and why there is a difference between inertial and accelerated motion (a body moving by inertia is represented by a straight timelike worldline, whereas the worldline of an accelerated body is curved). Minkowski found it necessary to stress that "Especially the concept of *acceleration* acquires a sharply prominent character" (this volume). This sharply prominent character of the acceleration comes from the *absolute* geometric property of the worldline of an accelerated body – the worldline of such a body is curved (deformed); therefore the absoluteness of acceleration merely reflects the absolute fact that the worldline an accelerating body is curved (deformed) and does not imply an absolute space with respect to which the body accelerates.

- Minkowski's four-dimensional physics allowed him not only to *explain* the physical meaning of length contraction, but to realize clearly that, exactly like the relativity principle, that effect is also a manifestation of the four-dimensionality of the world.

- In his four-dimensional physics Minkowski found that pairs of ordinary mechanical quantities are in fact space and time components of four-dimensional vectors and the ordinary electromagnetic quantities are components of new types of four-dimensional structures.

Einstein won the race with his mathematics professor Minkowski (of the existence of which neither of them suspected) and first published his special relativity in 1905 in which he postulated the equivalence of t and t'. The realization of this equivalence took him many years and it came as a result of the persistent analysis of his thought experiment of racing a light beam. This thought experiment became a paradox for Einstein when he studied Maxwell's equations at the Polytechnic Institute in Zurich. In Maxwell's theory the velocity of light is a universal constant ($c = (\mu_0\epsilon_0)^{-1/2}$) which meant for Einstein (due to his trust in "the truth of the Maxwell-Lorentz equations in electrodynamics" and that they "should hold also in the moving frame of reference."[19]) that if he travelled almost at the speed of light (relative, say, to Earth), a beam of light would still move away from him at velocity c, which is in Einstein's own words "in conflict with the rule of addition of velocities we knew of well in mechanics"[20] Later Einstein acknowledged that "the germ of the special relativity theory was already present in that paradox"[21] and explained that his "solution was really for the very concept of time, that is, that time is not absolutely defined but there is an inseparable connection between time and the signal velocity. With this connection, the foregoing extraordinary difficulty could be thoroughly solved. Five weeks after my recognition of this, the present theory of special relativity was completed."[22]

Einstein's realization that inertial observers in relative motion have different times had been accomplished through conceptual analyses *á la* Galileo. The development of this powerful method had later helped Einstein to make one of the greatest discoveries in the intellectual history of our civilization – that gravitational phenomena are not caused by gravitational forces but are a manifestation of the non-

[19] A. Pais, *Subtle Is the Lord: The Science and the Life of Albert Einstein* (Oxford University Press, Oxford 2005) p. 139

[20] A. Pais, Ibid.

[21] A. Folsing, *Albert Einstein: A Biography* (Penguin Books, New York 1997) p. 166

[22] A. Pais, Ibid.

Euclidean geometry of spacetime. However, in 1905 Einstein still did not understand fully all implications of his major discovery that t and t' should be treated equally. As a result, at that time and at least in the following several years Einstein did not have complete understanding of the above list of issues which Minkowski clarified in 1907 and 1908. For example, unlike Minkowski Einstein had to postulate the relativity principle without being able to explain its physical meaning. He also simply stated that the luminiferous ether was superfluous without any explanation, that is, he merely postulated that absolute motion does not exist. Einstein did not have the correct understanding of the *physical meaning* of length contraction either since at that time he had not yet fully understood and adopted Minkowski's four-dimensional physics.

One of the indications that Einstein did not fully comprehend the implications of the fact that observers in relative motion have different times is the very name of his theory – the theory of *relativity*.[23] Einstein believed that time is relative, whereas Minkowski *explained* the physical meaning of that relativity – observers in relative motion have *different times*. And Minkowski demonstrated that that relativity of time (and space as Minkowski first showed) is a manifestation of (or implies) an *absolute* four-dimensional world. So the essence of the "theory of relativity" is the discovery that reality is an *absolute* world – Minkowski's four-dimensional world; this world (spacetime) is absolute because it is not frame- or observer-dependent – spacetime is the *same* for all observers. Minkowski's explanation showed that *relative* quantities are manifestations of an underlying

[23] Sommerfeld specifically stressed this fact: "the widely misunderstood and not very fortunate name of "theory of relativity"." A. Sommerfeld, "To Albert Einstein's Seventieth Birthday." In: *Albert Einstein: Philosopher-Scientist.* P. A. Schilpp, ed., 3rd ed. (Open Court, Illinois 1969) pp. 99-105, p. 99.

absolute entity – space and time are relative to observers because observers in relative motion have different spaces and times when they use their ordinary three-dimensional language to represent the absolute spacetime, which is not divided in spaces and times (that is why no relativity of space and time is possible in a three-dimensional world).

What is even worse, is that Einstein insisted on *relativity* as the core concept of his theories and called his revolutionary theory of gravitation *the general theory of relativity*, which is a further indication of his slow acceptance of Minkowski's four-dimensional physics. As Synge remarked[24] Minkowski "protested against the use of the word 'relativity' to describe a theory based on an 'absolute' (space-time), and, had he lived to see the general theory of relativity, I believe he would have repeated his protest in even stronger terms."

It is well known that Einstein was "for general reasons, firmly convinced that there does not exist absolute motion"[25] and that Einstein regarded all motion as relative mostly due to Mach. And indeed Einstein kept the term "relativity" in his general theory because he believed that in that theory acceleration should also be treated as relative. In his 1914 paper *The Formal Foundation of the General Theory of Relativity*[26] Einstein repeated and extended Mach's argument for a relative acceleration. This fact alone is sufficient to demonstrate that even in 1914 Einstein had not fully understood Minkowski's spacetime physics.[27] As

[24] J. L. Synge, *Relativity: the general theory* (North-Holland, Amsterdam 1960) p. IX.

[25] A. Pais, loc. cit., p. 172.

[26] *The Collected Papers of Albert Einstein, Volume 6: The Berlin Years: Writings, 1914-1917* (Princeton University Press, Princeton 1997) p. 31.

[27] However, later in his life Einstein seems to have fully realized the implications of spacetime not only for physics but for our entire worldview as well (see last section). Regarding Mach, Einstein wrote in 1954: "As a matter of fact, one should no longer speak of Mach's

indicated above Minkowski particularly pointed out the prominent character of the concept of acceleration since the acceleration's absoluteness comes from the absolute fact that the worldline of an accelerating body is curved (deformed). It is true that Minkowski's explanation of the absoluteness of acceleration was given for the case of flat spacetime, whereas in 1914 Einstein was completing his theory of general relativity. However, the situation regarding the absoluteness of acceleration is exactly the same in the case of curved spacetime (i.e. in general relativity) – a body moving by inertia is represented by a geodesic worldline (which is the analog of a straight worldline in curved spacetime since it is curved only due to the curvature of spacetime, but is not additionally curved, i.e. it is not deformed), whereas an accelerating body is represented by a deformed (non-geodesic) worldline. Therefore acceleration in both flat and curved spacetime is absolute which demonstrate that Mach's view of relative acceleration is clearly wrong. Here is a concrete example to see why this is so. Mach argued that one could not say anything about the state of motion of a *single* particle in the Universe since he believed that one can talk only about motion *relative* to another body. However, that situation is crystal clear in Minkowski's spacetime physics – the worldline of a single particle in the Universe is either geodesic or deformed, which means that the particle is either moving by inertia or accelerating.

Despite the difficulties Einstein had had with understanding and adopting Minkowski's spacetime physics, the mastering of the method of conceptual analyses involving thought experiments helped him draw all three-dimensional implications of the equivalence of the times of observers in relative motion. For example, the thought experiments led Einstein to the relation between mass and energy $E = mc^2$

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principle at all" (A. Pais, loc. cit., p. 288).

which now bears his name although it was discovered before him in the framework of the electron theory.[28]

In view of all these facts it is inexplicable how could anyone say that Minkowski had not understood Einstein's 1905 paper on special relativity. I will give two examples which are even more inexplicable since they come from the authors of two very informative and otherwise excellent papers.

In 1979 Galison[29] wrote: "At this early time (1907) it is clear that Minkowski did not understand the import of Einstein's theory." As we have seen the actual situation had been just the opposite. Galison had in mind Minkowski's enthusiasm for arriving at an electromagnetic picture of the world based on his world postulate and the electron theory as suggested by the last paragraph of *Space and Time* (this volume): "The validity without exception of the world postulate is, I would think, the true core of an electromagnetic world view which, as Lorentz found it and Einstein further unveiled it, lies downright and completely exposed before us as clear as daylight." First, not only in 1907 but also in 1908 (when *Space and Time* was presented in Cologne) Minkowski had the same view; moreover his Cologne lecture essentially explained in a non-technical language the main results of his lecture given on December 21, 1907. And I do not see anything wrong with Minkowski's hope for a unified world picture; at that time the other fundamental interactions were unknown, so it was perfectly natural to try to find a unified picture of the world on the basis of

[28]When it was initially derived in the electron theory, that expression contained the famous factor of 4/3, which was later accounted for; see V. Petkov, *Relativity and the Nature of Spacetime*, 2nd ed. (Springer, Heidelberg 2009) Chap. 9, particularly Sec. 9.3 and the references therein.

[29]P. L. Galison, Minkowski's Space-Time: From Visual Thinking to the Absolute World, Historical Studies in the Physical Sciences, Vol. 10 (1979) pp. 85-121, p. 93.

what was known. Most important, however is the following. If "Minkowski did not understand the import of Einstein's theory" because he was positively looking at the electron theory, then by exactly the same argument Einstein did not understand the import of his own theory. In January 1909 Einstein wrote[30] "In conclusion, I would also like to point to the importance of the recently published paper by Ph. Frank, which, by taking into account the Lorentz contraction, restores the agreement between Lorentz's treatment, based on the electron theory, and Minkowski's treatment of the electrodynamics of moving bodies. The advantage of the treatment based on the electron theory consists, on the one hand, in providing a graphic interpretation of the field vectors and, on the other hand, in dispensing with the arbitrary assumption that the derivatives of the velocity of matter do not appear in the differential equations." As seen from this quote, in 1909 Einstein viewed "Minkowski's treatment of the electrodynamics of moving bodies" as different from Lorentz' treatment "*based on the electron theory*" and pointed out the "advantage of the treatment based on the electron theory."

Now the prevailing view is that the electron theory was wrong. I am afraid that that is rather a simplistic view. It is now clear what in the electron theory was undoubtedly wrong – e.g. the electron is not a small charged sphere. A completely wrong theory cannot make a number of correct predictions – e.g. the electron theory predicted that the electron mass increases as the electron's velocity increases *before* the theory of relativity, yielding the correct velocity dependence, and that the relation between energy and mass is $E = mc^2$. That is why it is maybe more appropriate to say

[30] A. Einstein, Comment on the paper of D. Mirimanoff "On the Fundamental Equations..." *Annalen der Physik* **28** (1909) pp. 885-888. In: *The Collected Papers of Albert Einstein, Volume 2: The Swiss Years: Writings, 1900-1909* (Princeton University Press, Princeton 1989), p. 356.

that today "the state of the classical electron theory reminds one of a house under construction that was abandoned by its workmen upon receiving news of an approaching plague. The plague in this case, of course, was quantum theory. As a result, classical electron theory stands with many interesting unsolved or partially solved problems."[31]

Unfortunately, exactly a hundred years after Minkowski's lecture *Space and Time* Damour[32] wrote: "First, I would like (after many others. . .) to stress that Minkowski probably did not really comprehend the conceptual novelty of Einstein's June 1905 paper on Special Relativity, and especially the results therein concerning *time*. Indeed, in his Cologne lecture Minkowski says that, while Einstein "deposed [time] from its high seat", "neither Einstein nor Lorentz made any attack on the concept of space. . . " However, this was precisely one of the key new insights of Einstein, namely the *relativity of simultaneity*!"

Now, *thanks to Minkowski*, we know that relativity of simultaneity does imply many spaces since *a space constitutes a class of simultaneous events* – two observers in relative motion have different classes of simultaneous events and therefore different spaces and vice versa (as Minkowski discovered two observers in relative motion have different spaces and therefore different classes of simultaneous events). However, in 1905 Einstein was *totally unaware* of this. He had been occupied with the idea of time and how to *measure* times and distances. Even a quick look at how Einstein arrived at the idea of relativity of simultaneity in his 1905 paper shows that he did that in an *operational way* – by analyzing the *procedure* of synchronizing distant clocks through light

[31]P. Pearle, Classical Electron Models. In: *Electromagnetism: Paths to Research*, ed. by D. Teplitz (Plenum Press, New York 1982) pp. 211-295, p. 213.

[32]T. Damour, "What is missing from Minkowski's "Raum und Zeit" lecture", *Annalen der Physik.* **17**, No. 9-10, (2008) pp. 619-630, p. 627.

signals; relativity of simultaneity follows immediately from the fact that the velocity of light is c for all observers. That is why Einstein *himself* had never claimed that he had realized that observers in relative motion have different spaces. On the contrary, as indicated above *three years after* his 1905 paper (in May 1908) he reacted negatively towards the introduced by Minkowski absolute four-dimensional world and therefore negatively towards the very idea of many spaces since *it was the idea of many spaces* that led Minkowski to the absolute four-dimensional world. As we saw above Minkowski's geometrical approach helped him to realize first that as observers in relative motion have different times they necessarily must have different spaces as well, and then he had probably immediately seen that many spaces imply an absolute four-dimensional world.

As unfounded as the statement above (that Einstein had discovered that observers in relative motion have different spaces), is another statement in Damour's article:

> In addition, when Minkowski introduces the (geometrically motivated) concept of proper time, he does not seem to fully grasp its physical meaning. However, this is the second key new insight brought in by Einstein concerning time, namely the fact (explicitly discussed by Einstein) that, when comparing a moving clock to one remaining at rest (and marking the corresponding 'rest' coordinate time t), the moving clock will mark (upon being reconvened with the sedentary clock) the time
>
> $$\tau = \int dt\sqrt{1 - v^2/c^2}$$
>
> i.e. Minkowski's proper time. It seems that Minkowski was not aware of this.

Minkowski was certainly aware of this expression *without the integral* (there is no integral in Einstein's paper as Damour admits but in a footnote) – on October 9, 1907 he wrote to Einstein to request a copy of his 1905 paper.[33] Damour's suggestion that Minkowski might have misread the paper – "This is another example of a scientist misreading a paper which he knew, however, to be central to his research topic!"[34] – seems virtually impossible since "Minkowski had written to Einstein asking for a reprint of his 1905 paper, in order to study it in his joint seminar with Hilbert"[35] (could Minkowski have misread a *key* paper that had been *studied* at the seminar he co-directed with Hilbert?).

What is most important, however, is that, like the above issue of many spaces, Damour again seems to read more in Einstein's 1905 paper. Einstein had completed that paper only five weeks after he had realized the equivalence of the times of observers in relative motion and had been still struggling with its consequences. By contrast, Minkowski seems to have had more than two years to explore those consequences – Minkowski appears ho have realized independently the equivalence of the times of observers in relative motion almost certainly as late as the summer of 1905.[36] The best proof that Minkowski fully understood

[33] S. Walter, Minkowski, Mathematicians, and the Mathematical Theory of Relativity, in H. Goenner, J. Renn, J. Ritter, T. Sauer (eds.), *The Expanding Worlds of General Relativity*, Einstein Studies, volume 7, (Birkhäuser, Basel 1999) pp. 45-86, p. 47.

[34] T. Damour, loc. cit., p. 627.

[35] L. Corry, Hermann Minkowski and the Postulate of Relativity, *Archive for History of Exact Sciences* **51** (1997) p. 273-314, p. 276.

[36] Let me repeat here the two indications of that, mentioned above, which cannot be merely ignored. First, Born's recollections quoted in the first section; there is no reason whatsoever to suspect that Born would invent such recollections. Second, what is far more important, however, is the full-blown four-dimensional formalism of "four-dimensional physics," which Minkowski reported on December 21, 1907, and the depth of his understanding of the electrodynamics

the physical meaning of proper time (which is quite natural given that this concept was introduced by himself) is the fact that the modern introduction and definition of proper time is *identical* to that of Minkowski. Only an in-depth and complete understanding of the new concepts of space and time and their union made their introduction and definition so precise that they remained unchanged more than a hundred years later. As this should be self-evident since it was Minkowski who thoroughly developed these new concepts it is inexplicable why not only did Damour make the above claim but found it necessary to repeat it: "Minkowski did not fully grasp the physical meaning of what he was doing."[37]

Minkowski's understanding of the physical meaning of time and spacetime had been so deep that with the introduction of proper time he essentially demonstrated that an observer should use *two* times in his rest frame – *proper* and *coordinate* times (τ and t) – which provided the correct physical treatment of time (i) in accelerated reference frames in special relativity, and later (ii) in general relativity. Minkowski did not call the time t coordinate time, but the presence of the *two* times in the *same* reference frame is obvious from the way he defined proper time (this volume):

$$d\tau = \frac{1}{c}\sqrt{c^2dt^2 - dx^2 - dy^2 - dz^2}.$$

The expression $c^2dt^2 - dx^2 - dy^2 - dz^2$ is the interval (the spacetime distance) ds^2 (in a reference frame) between the two infinitesimally close events on the worldline of a particle; the length of the worldline between these events is the proper time $d\tau$. If the particle's worldline is straight, which means that the particle moves with constant velocity, in

of moving bodies and the absolute four-dimensional world; such a revolution in both physics and mathematics could not have been possible if he had merely developed others' ideas.

[37] T. Damour, loc. cit., p. 627.

its inertial reference frames proper and coordinate times coincide. However, if the particle accelerates, its worldline is curved and an observer in the particle's accelerating frame should use both proper and coordinate times.

If Damour had insisted on keeping in his paper the repeated unfortunate expression "did not fully grasp the physical meaning of what he was doing," he should have used it for Einstein's understanding of the physical meaning of the time (in the case discussed by Damour) which Minkowski later called proper time (but that would have been equally unfair since as indicated above Einstein completed his 1905 paper only five weeks after his profound insight that the times of observers in relative motion should be treated equally). In the above calculation quoted by Damour, Einstein determined the time of a clock in *circular* motion: "If there are two synchronous clocks in A, and one of them is moved along a closed curve with constant velocity[38] until it has returned to A, which takes, say, t sec, then this clock will lag on its arrival at A $\frac{1}{2}t(v/V)^2$ sec behind the clock that has not been moved."[39] Einstein arrived at this result by using the Lorentz transformation of the times of two inertial clocks in relative motion, which generally deals with *coordinate* time. As coordinate and proper time coincide in inertial reference frames (moving with constant velocity) no misunderstanding is likely. But in an accelerating reference frame coordinate and proper time do not coincide. When Einstein compared the times of the *accelerating* clock (moving along the closed curve) and the stationary clock he used what was later called the proper time of the accelerating clock

[38] Even in the new translation of Einstein's 1905 paper the German word *Geschwindigkeit* has been again erroneously translated in this sentence as *velocity.* Obviously, the velocity of the clock along a closed curve is not constant; what is constant is the clock's speed.

[39] A. Einstein, On the electrodynamics of moving bodies, *The Collected Papers of Albert Einstein, Volume 2: The Swiss Years: Writings, 1900-1909* (Princeton University Press, Princeton 1989), p. 153.

without having any idea that that time is a *second* time in the reference frame of the accelerating clock, which is *different* from the coordinate time (Minkowski reported the concept of proper time more than two years later; when he arrived at that concept is unclear, most probably not so long after the seminar held in the summer of 1905).

Damour further wrote[40] that Minkowski "had (seemingly) not fully grasped the striking result of Einstein that proper time along any polygonal (or curved) time-like line between two points in spacetime is smaller than the proper time along the straight line joining the two points. If he had realized it clearly, he would have commented that this is just the opposite of the usual triangular inequality." First, the wording of "the striking result of Einstein that proper time..." is inappropriate – it is well known and indicated above that in 1905 Einstein could not have had any idea of what proper time is. Second, as Minkowski defined proper time as length along a timelike worldline he knew perfectly what proper time is, and it is indeed a valid question why he did not define the triangle inequality in spacetime as well.

I think the most probable explanation is that since he had been completely occupied with developing the spacetime physics and its four-dimensional mathematical formalism his first priority had been (as seen from his three papers) the electrodynamics of moving bodies. The work on the kinematical consequences of the absolute four-dimensional world (e.g. the special role of acceleration stressed by Minkowski) had been scheduled for later as Minkowski clearly alluded to such a plan: "The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines" (this volume). The triangle inequality is clearly

[40] T. Damour, loc. cit., p. 629.

such an interrelation between worldlines.

To expect more from someone who had already done so much for such a short period of time, and who would have indisputably done even more, if he had not been taken away from us when he was at the peak of his intellectual strength, is very unfair.

It is important to stress that after his initial hostile attitude towards Minkowski's spacetime physics Einstein gradually adopted it since it was essential for his general relativity. In 1946 in his Autobiography Einstein summarized Minkowski's main contribution:[41]

> Minkowski's important contribution to the theory lies in the following: Before Minkowski's investigation it was necessary to carry out a Lorentz-transformation on a law in order to test its invariance under such transformations; he, on the other hand, succeeded in introducing a formalism such that the mathematical form of the law itself guarantees its invariance under Lorentz-transformations. By creating a four-dimensional tensor-calculus he achieved the same thing for the four-dimensional space which the ordinary vector-calculus achieves for the three spatial dimensions. He also showed that the Lorentz-transformation (apart from a different algebraic sign due to the special character of time) is nothing but a rotation of the coordinate system in the four-dimensional space.

As seen from his estimation of Minkowski's contribution Einstein did not explicitly credit Minkowski for

[41] A. Einstein, "Autobiographical notes." In: *Albert Einstein: Philosopher-Scientist.* Paul A. Schilpp, ed., 3rd ed. (Open Court, Illinois 1969) pp. 1-94, p. 59.

demonstrating that the relativity postulate and length contraction imply an absolute four-dimensional world; we will return to this point in the last section. On the other hand, Einstein credited Minkowski for showing that the Lorentz transformations are rotations in spacetime, whereas it was Poincaré who first published that result in 1906.[42]

Let me stress it one more time – Einstein's achievements speak for themselves, so no one can downplay his contributions. I think Minkowski's four-dimensional (spacetime) physics and Einstein's discovery that gravity is a manifestation of the spacetime curvature will forever remain as the two greatest intellectual achievements. The approaches of Minkowski and Einstein are distinctly different, but they both proved to be so extraordinarily productive that should become integral parts of the way of thinking of any scientist who works on the front line of research in any field. Minkowski's and Einstein's proven but not fully studied approaches form the core of a research strategy that has being developed and employed at the Minkowski Institute (http://minkowskiinstitute.org/).

In addition, I have a personal reason not to even think of downplaying Einstein's contributions. I have always admired him for the way he arrived at his two theories – by employing and extending Galileo's way of doing physics through conceptual analyses and thought experiments. Moreover, my own way of thinking about physical phenomena was consciously formed by studying the methods of great physicists which led them to groundbreaking discoveries, particularly those of Galileo and Einstein; much later I discovered and started to appreciate thoroughly Minkowski's approach to physics.

Also, I fully share Einstein's firm position that quantum mechanics does not provide a complete description of the

[42] H. Poincaré, Sur la dynamique de l'électron. *Rendiconti del Circolo matematico Rendiconti del Circolo di Palermo* **21** (1906) pp.129-176.

quantum world in a sense that it does not contain a spacetime model of the quantum object *itself*. I believe a theory that describes only the *state* of something, not the something itself, is intrinsically incomplete. As now no one can seriously question the probabilistic nature of quantum phenomena it appears easily tempting to state that Einstein's intuition that God does not play dice was wrong. I think such a temptation will remain baseless until we understand what the quantum object is.

Leaving aside the issue of whether God would care about a human's opinion on how he should behave, just imagine the following (very probable in my view) development in quantum physics, which may reveal an unanticipated meaning of Einstein's intuition. As Galileo's and Einstein's conceptual analyses (which proved to be physics at its best) are now almost explicitly regarded as old-fashioned (no leading physics journal would publish a paper containing a deep conceptual analysis of an open question), it is not surprising that the so called quantum paradoxes remained unresolved almost a century after the advent of quantum mechanics.

Despite Feynman's desperate appeal to regard Nature as absurd[43] the history of science teaches us that all apparent paradoxes are caused by some implicit assumptions. A consistent conceptual analysis of only one of those quantum mechanical paradoxes – say, the famous double-slit experiment, discussed by Feynman – almost immediately identifies an implicit assumption[44] – we have been taking for granted that quantum objects exist *continuously* in time although there has been nothing either in the experimental

[43]Feynman wrote: "The theory of quantum electrodynamics describes Nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you can accept Nature as She is – absurd." R. P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton University Press, Princeton 1985) p. 10.

[44]V. Petkov, *Relativity and the Nature of Spacetime*, 2nd ed. (Springer, Heidelberg 2009) Chap. 10.

evidence or in the theory that compels us to do so. Just imagine – a fundamental *continuity* (continuous existence in time) at the heart of quantum physics. And no wonder that such an implicit assumption leads to a paradox – an electron, for example, which is always registered as a pointlike entity and which exists continuously in time, is a classical particle (i.e. a worldline in spacetime) that cannot go through both slits in the double-slit experiment in order to form an interference pattern.[45]

However, if we abandon the implicit assumption and replace it explicitly with its alternative – discontinuous existence in time – the paradox disappears. Then an electron is, in the ordinary three-dimensional language, an ensemble[46] of constituents which appear-disappear $\sim 10^{20}$ times per second (the Compton frequency). Such a quantum object can pass simultaneously through *all* slits at its disposal.

In Minkowski's four-dimensional language (trying to extract more from his treasure), such an electron is not a worldline but a "disintegrated" worldline whose worldpoints are scattered all over the spacetime region where the electron wavefunction is different from zero. Such a model of the quantum object and quantum phenomena in general provides a surprising insight into the physical meaning of probabilistic phenomena in spacetime – an electron is a *probabilistic* distribution of worldpoints which is *forever given* in spacetime.

Had Minkowski lived longer he might have described such a spacetime picture by the mystical expression "predetermined probabilistic phenomena." And, I guess,

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[45] Double-slit experiments with *single* electrons and photons prove that an interference pattern is observed, which is only possible if every single electron or photon goes through both slits.

[46] A. H. Anastassov, Self-Contained Phase-Space Formulation of Quantum Mechanics as Statistics of Virtual Particles, *Annuaire de l'Universite de Sofia "St. Kliment Ohridski", Faculte de Physique* **81** (1993) pp. 135-163.

Einstein would be also satisfied – God would not play dice since a probabilistic distribution in spacetime exists eternally there.

3. Minkowski and Poincaré

This section was the most difficult to write since I have not found any clue of how Minkowski would have explained the obvious fact – that Poincaré was not mentioned in his Cologne lecture *Space and Time*. Minkowski was certainly aware of Poincaré's paper *Sur la dynamique de l'électron* published in 1906 (but received by *Rendiconti del Circolo matematico Rendiconti del Circolo di Palermo* on July 23, 1905) since he quoted it in his previous lectures given in November and December 1907. In his paper Poincaré first published the important result that the Lorentz transformations had a geometric interpretation as rotations in what he seemed to have regarded as an *abstract* four-dimensional space with time as the fourth dimension.[47]

Here are two attempts to explain Minkowski's omission to mention Poincaré's paper in his Cologne lecture.

> In the absence of any clear indication why Minkowski left Poincaré out of his lecture, a speculation or two on his motivation may be entertained. If Minkowski had chosen to include some mention of Poincaré's work, his own contribution may have appeared derivative. Also, Poincaré's modification of Lorentz's theory of electrons constituted yet another example of the cooperative role played by the mathematician in the elaboration of physical theory. Poincaré's "more mathematical" study of Lorentz's electron

[47] H. Poincaré, Sur la dynamique de l'électron, *Rendiconti del Circolo matematico Rendiconti del Circolo di Palermo* **21** (1906) pp. 129-176.

> theory demonstrated the mathematician's dependence upon the insights of the theoretical physicist, and as such, it did little to establish the independence of the physical and mathematical paths to the Lorentz group. The metatheoretical goal of establishing the essentially mathematical nature of the principle of relativity was no doubt more easily attained by neglecting Poincaré's elaboration of this principle.[48]

> My conjecture is that Minkowski, helped by his background reading of some of the works of Lorentz and Poincaré (which, however, *did not include* their most recent contributions of 1904-1905...) had discovered by himself, in the summer of 1905 (without knowing about the 1905 papers of Poincaré) the fact that Lorentz transformations preserve the quadratic form $-c^2t^2 + x^2(+y^2 + z^2)$. If that reconstruction is correct, he must have been all the more eager, when he later realized that he had been preceded by Poincaré, to find reasons for downplaying Poincaré's work.[49]

I think one should also ask why in 1946 in his Autobiography[50] (as quoted in Section 2) Einstein wrote that Minkowski "showed that the Lorentz-transformation [...] is

[48] S. Walter, Minkowski, Mathematicians, and the Mathematical Theory of Relativity, in H. Goenner, J. Renn, J. Ritter, T. Sauer (eds.), *The Expanding Worlds of General Relativity*, Einstein Studies, volume 7, (Birkhäuser, Basel 1999) pp. 45-86, p. 58.

[49] T. Damour, What is missing from Minkowski's "Raum und Zeit" lecture, *Annalen der Physik.* **17**, No. 9-10, (2008) pp. 619-630, p. 626.

[50] A. Einstein, "Autobiographical notes." In: *Albert Einstein: Philosopher-Scientist.* Paul A. Schilpp, ed., 3rd ed. (Open Court, Illinois 1969) pp. 1-94, p. 59.

nothing but a rotation of the coordinate system in the four-dimensional space." It seems Einstein was either unaware in 1946 (which is highly unlikely) of the fact that it was Poincaré who first published that result, or he knew (perhaps from Born) that Minkowski independently had made the same discovery.

Another interesting fact is that not someone else but a famous *French* physicist credited Minkowski for the discovery of spacetime. In 1924 Louis de Broglie wrote in his doctoral thesis *Recherches sur la théorie des quanta*:[51] "Minkowski showed first that one obtains a simple geometric representation of the relationships between space and time introduced by Einstein by considering an Euclidean manifold of 4 dimensions called Universe or spacetime." Another contemporary French physicist – Thibault Damour (quoted above) – also thinks that "the replacement of the separate categories of space and time with the new physical category of space-time is [...] more properly attributed to Hermann Minkowski and not to Poincaré."[52]

Probably we will never learn why Minkowski did not quote Poincaré in his lecture *Space and Time* in 1908. However, a

[51]"Minkowski a montré le premier qu'on obtenait une représentation géométrique simple des relations de l'espace et du temps introduites par Einstein en considérant une multiplicité euclidienne à 4 dimensions dite Univers ou Espace-temps," Louis de Broglie, *Recherches sur la théorie des quanta*, Réédition du texte de 1924. (Masson, Paris 1963), p. 27. Strangely, the word "appears" (which is clearly not in the original French text) had been inserted into the sentence translated into English by Kracklauer: "Minkowski appears to have been first to obtain a simple geometric representation of the relationships introduced by Einstein between space and time consisting of a Euclidian 4-dimensional space-time," Louis-Victor de Broglie, *On the Theory of Quanta*, translated by A. F. Kracklauer (2004); available at the website of *Annales de la Fondation Louis de Broglie* (http://aflb.ensmp.fr/LDB-oeuvres/De_Broglie_Kracklauer.htm).

[52]T. Damour, *Once Upon Einstein*, Translated by E. Novak (A. K. Peters, Wellesley 2006) p. 49.

similar question applies to Poincaré himself: "In the lecture Poincaré delivered in Göttingen on the new mechanics in April 1909, he did not see fit to mention the names of Minkowski and Einstein."[53] Poincaré could have used the fact that his lecture was only around three months after Minkowski's death to credit Minkowski for fully developing the four-dimensional physics based on the idea of spacetime which Poincaré first published.

I think the discovery of spacetime is a doubly sad story. First, unlike Minkowski, Poincaré seems to have seen nothing revolutionary in the idea of a mathematical four-dimensional space as Damour remarked[54] – "although the first discovery of the mathematical structure of the space-time of special relativity is due to Poincaré's great article of July 1905, Poincaré (in contrast to Minkowski) had never believed that this structure could really be important for physics. This appears clearly in the final passage that Poincaré wrote on the question some months before his death":

> Everything happens as if time were a fourth dimension of space, and as if four-dimensional space resulting from the combination of ordinary space and of time could rotate not only around an axis of ordinary space in such a way that time were not altered, but around any axis whatever...
>
> What shall be our position in view of these new conceptions? Shall we be obliged to modify our conclusions? Certainly not; we had adopted a convention because it seemed convenient and we had said that nothing could constrain us

[53] S. Walter, Minkowski, Mathematicians, and the Mathematical Theory of Relativity, in H. Goenner, J. Renn, J. Ritter, T. Sauer (eds.), *The Expanding Worlds of General Relativity*, Einstein Studies, volume 7, (Birkhäuser, Basel 1999) pp. 45-86, p. 57.

[54] T. Damour, loc. cit., p. 51.

> to abandon it. Today some physicists want to adopt a new convention. It is not that they are constrained to do so; they consider this new convention more convenient; that is all. And those who are not of this opinion can legitimately retain the old one in order not to disturb their old habits. I believe, just between us, that this is what they shall do for a long time to come.[55]

Poincaré believed that our physical theories are only *convenient descriptions* of the world and therefore it is really a matter of *convenience* and *our choice* which theory we would use. As Damour stressed it, it was "the sterility of Poincaré's scientific philosophy: complete and utter "conventionality" [...] which stopped him from taking seriously, and developing as a physicist, the space-time structure which he was the first to discover."[56]

What makes Poincaré's failure to comprehend the profound physical meaning of the relativity principle and the geometric interpretation of the Lorentz transformations especially sad is that it is perhaps the most cruel example in the history of physics of how an inadequate philosophical

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[55] H. Poincaré, *Mathematics and Science: Last Essays (Dernières Pensées)*, Translated by J.W. Bolduc (Dover, New York 1963) pp. 23-24. Poincaré even appeared to have thought that the spacetime convention would not be advantageous: "It quite seems, indeed, that it would be possible to translate our physics into the language of geometry of four dimensions. Attempting such a translation would be giving oneself a great deal of trouble for little profit, and I will content myself with mentioning Hertz's mechanics, in which something of the kind may be seen. Yet, it seems that the translation would always be less simple than the text, and that it would never lose the appearance of a translation, for the language of three dimensions seems the best suited to the description of our world, even though that description may be made, in case of necessity, in another idiom." H. Poincaré, *Science and Method*, In: *The Value of Science: Essential Writings of Henri Poincaré* (Modern Library, New York 2001) p. 438.

[56] T. Damour, loc. cit., p. 52.

position can prevent a scientist, even as great as Poincaré, from making a discovery. However, this sad example can serve a noble purpose. Science students and young scientists can study it and learn from it because scientists often think that they do not need any philosophical position for their research:

> Scientists sometimes deceive themselves into thinking that philosophical ideas are only, at best, decorations or parasitic commentaries on the hard, objective triumphs of science, and that they themselves are immune to the confusions that philosophers devote their lives to dissolving. But there is no such thing as philosophy-free science; there is only science whose philosophical baggage is taken on board without examination.[57]

Second, it seems virtually certain that Minkowski independently arrived at two important results – (i) the equivalence of the times of observers in relative motion and (ii) the fact that the Lorentz transformations preserve the quadratic form $c^2t^2-x^2-y^2-z^2$ and can therefore be regarded geometrically as rotation in a four-dimensional space with time as the fourth dimension. But these results were first published by Einstein and Poincaré, respectively. As indicated in Section 2 the best proof that Minkowski, helped by his extraordinary geometrical imagination, had made these discoveries independently of Einstein and Poincaré, is the introduced by him four-dimensional (spacetime) physics with a fully developed mathematical formalism and his deep understanding of the new worldview and its implications. Born's recollections given in Section 2 only confirm what follows from a careful study of Minkowski's results.

[57] D. C. Dennett, *Darwin's Dangerous Idea: Evolution and the Meanings of Life* (Simon and Schuster, New York 1996) p. 21.

4. Minkowski and gravitation

On January 12, 1909 only several months after his Cologne lecture *Space and Time* at the age of 44 Minkowski tragically and untimely departed from this strange world (as Einstein would call it later). We will never know how physics would have developed had he lived longer.

What seems undeniable is that the discovery of the true cause of gravitation – the non-Euclidean geometry of spacetime – would have been different from what actually happened. On the one hand, Einstein's way of thinking based on conceptual analyses and thought experiments now seems to be the only way powerful enough to decode the unimaginable nature of gravitation. However, on the other hand, after Minkowski had written the three papers on relativity included here, he (had he lived longer) and his friend David Hilbert might have formed an unbeatable team in theoretical physics and might have discovered general relativity (surely under another name) before Einstein.

As there is no way to reconstruct what might have happened in the period 1909-1915 I will outline here what steps had been logically available to Minkowski on the basis of his results. Then I will briefly discuss whether their implications would lead towards the modern theory of gravitation – Einstein's general relativity.

In 1907 (most probably in November) Einstein had already been well ahead of Minkowski when he made a gigantic step towards the new theory of gravity:[58]

> I was sitting in a chair in the patent office at Bern when all of a sudden a thought occurred to me: "If a person falls freely he will not feel his own weight." I was startled. This simple thought

[58]Quoted from: A. Pais, *Subtle Is the Lord: The Science and the Life of Albert Einstein* (Oxford University Press, Oxford 2005) p. 179.

made a deep impression on me. It impelled me toward a theory of gravitation.

In the first row of this photograph (probably taken around 1905) are Minkowski (left) David Hilbert's wife, Käthe, and David Hilbert. Source: D. E. Rowe, A Look Back at Hermann Minkowski's Cologne Lecture "Raum und Zeit," *The Mathematical Intelligencer*, Volume 31, Number 2 (2009), pp. 27-39.

Einstein had been so impressed by this insight that he called it the "happiest thought" of his life.[59] And indeed this is a crucial point – at that time Einstein had been the only human who realized that no gravitational force acted on a falling body. Then he struggled eight years to come up with a theory – his general relativity – according to which gravity is not a force but a manifestation of the curvature of spacetime.

Here I will stress particularly the core of general relativity which reflects Einstein's "happiest thought" – the geodesic hypothesis according to which a falling particle is not subject to a gravitational force. In other words, the geodesic

[59] A. Pais, Ibid.

hypothesis in general relativity assumes that the worldline of a free particle is a timelike *geodesic* in spacetime. The geodesic hypothesis is regarded as "a natural generalization of Newton's first law,"[60] that is, "a mere extension of Galileo's law of inertia to curved spacetime."[61] This means that *in general relativity a particle, whose worldline is geodesic, is a free particle which moves by inertia.*

The geodesic hypothesis has been *confirmed* by the experimental fact that particles falling towards the Earth's surface *offer no resistance to their fall* – a falling accelerometer, for example, reads zero resistance (i.e. zero acceleration; the observed *apparent* acceleration of the accelerometer is caused by the spacetime curvature induced by the Earth; more precisely, in spacetime physics it is caused by geodesic deviation – the fact that there are no parallel worldlines in curved spacetime). The experimental fact that particles do not resist their fall (i.e. their apparent acceleration) means that they move by inertia and therefore no gravitational force is causing their fall. It should be emphasized that a gravitational force would be required to accelerate particles downwards *only if* the particles *resisted* their acceleration, because *only then* a gravitational force would be needed to *overcome* that resistance.

Let us now imagine how Minkowski would have approached the issue of gravitation. By analogy with Maxwell's electrodynamics he had already modified Newton's gravitational theory in order that the speed of gravity be equal to that of light c (Poincaré also proposed such a modification in his 1906 paper on the dynamics of the electron). Now, thanks to the genius of Einstein, we know that electromagnetism is fundamentally different from

[60] J. L. Synge, *Relativity: the general theory.* (Nord-Holand, Amsterdam 1960) p. 110.

[61] W. Rindler, *Relativity: Special, General, and Cosmological* (Oxford University Press, Oxford 2001) p. 178.

gravitation – electromagnetic phenomena are caused by electromagnetic *forces*, whereas gravitational phenomena are manifestation of the non-Euclidean *geometry of spacetime* which means that there are no gravitational forces in Nature.

The natural question is whether Minkowski would have found any reasons to revise his modified version of Newton's theory of gravity. Perhaps many physicists would say 'highly unlikely.' And they might be right. But looking at what Minkowski had achieved for so short a period of time, I think his genius should never be underestimated (even because that would constitute a contradiction in terms). Let us see what logical options Minkowski had after his third lecture *Space and Time*.

Minkowski had been aware of two relevant facts – (i) the motion of particles with constant velocity cannot be detected experimentally since the particles move non-resistantly, i.e. by inertia (in other words, an experiment always detects the *lack of resistance* of an inertial particle, and in this sense inertial motion is absolute or frame-independent), and (ii) the accelerated motion of a particle can be discovered experimentally since the particle *resists* its acceleration (so accelerated motion is also absolute in this sense and therefore frame-independent).

The accelerated motion had already been causing problems after the publication of Einstein's special relativity in 1905 since it appeared that the experimental detection of accelerated motion provided experimental support for the absolute space – if a particle's acceleration is *absolute* (since it is measurable), then such an acceleration is with respect to the absolute space, which contradicts both Einstein's special relativity and particularly Minkowski's interpretation of the relativity principle according to which observers in relative motion have different times and *spaces* (whereas an absolute space implies a *single* space).

However, Minkowski had not been concerned about such an apparent contradiction at all. He provided rigorous criteria for inertial and accelerated motion[62] – a free particle, which moves by inertia, is a straight timelike worldline in Minkowski spacetime, whereas the timelike worldline of an accelerating particle is clearly different – it is *curved* (i.e. *deformed*). That is why Minkowski wrote at the beginning of Section III of *Space and Time*: "Especially the concept of *acceleration* acquires a sharply prominent character."

These criteria show that in spacetime the absoluteness of inertial (non-resistant) and accelerated (resistant) motion become more understandable – the straightness of a timelike worldline (representing inertial motion) and the curvature or rather the *deformation* of a timelike worldline (representing accelerated motion) are absolute (frame-independent) properties of worldlines. These absolute properties of worldlines (straightness and deformation) correspond to the absoluteness (frame-independence) of inertial and accelerated motion in terms of experimental detection – it is an experimental fact that a particle moving by inertia offers no resistance to its uniform motion, and it is an experimental fact that an accelerating particle resists its acceleration.

Then, as indicated in Section 2, it becomes evident that absolute acceleration is a mere manifestation of the *deformation* of the worldline of an accelerating particle and *does not imply some absolute space with respect to which the particle accelerates*. Exactly in the same way, absolute inertial motion reflects the straightness of the worldline of an inertial particle and does not imply some absolute space with respect to which the particle moves with constant velocity.

Perhaps Minkowski knew all this well. What is more

[62] In the beginning of Section II of his paper *Space and Time* (this volume) Minkowski wrote: "a straight line inclined to the t-axis corresponds to a uniformly moving substantial point, a somewhat curved worldline corresponds to a non-uniformly moving substantial point."

important, however, is that he certainly knew that an accelerating particle is represented by a curved (deformed) worldline. Then he might have realized that inertia – the *resistance* a particle offers to its acceleration – could be regarded as arising from a four-dimensional stress[63] in the deformed worldline, or rather worldtube, of an accelerating particle. Certainly, Minkowski would have been enormously pleased with such a discovery because inertia would have turned out to be another manifestation of the four-dimensionality of the absolute world since only a *real* four-dimensional worldtube could resist its deformation (by analogy with an ordinary deformed rod which resists its deformation). Of course, the question of whether or not Minkowski could have noticed this surprising four-dimensional explanation of the origin of inertia (the origin of the resistance to acceleration) will forever remain unanswerable; but that explanation of inertia follows *logically* from the fact that an accelerating particle is a *deformed* worldtube and therefore would have been a legitimate logical option for Minkowski, especially given the fact that all his contributions to mathematics and physics demonstrated his innovative ability to explore the deep logical structure of what he studied.

We saw that Minkowski's spacetime criteria for inertial and accelerated motion spectacularly resolved the old (since Newton) question of the meaning of absolute acceleration – the acceleration of a particle is absolute not because it accelerates with respect to an absolute space, but because the particle's worldline is curved (deformed) which is an absolute geometric property. Then by asking the obvious question "What is the link between the two absolute properties of an accelerating particle – the absolute geometric property (the deformation of its worldline) and the absolute physical

[63] V. Petkov, *Relativity and the Nature of Spacetime*, 2nd ed. (Springer, Heidelberg 2009) Chap. 9.

property reflected in the fact that an accelerating particle resists its acceleration?" we are led to the surprising insight about the origin of inertia – the resistance a particle offers to its acceleration is in fact the static resistance in the deformed worldline of the accelerating particle.

To see even better the enormous potential of Minkowski's criteria for inertial and accelerated motion let us imagine two scenarios.

First, imagine that Minkowski or someone else who had had profound understanding of Minkowski's spacetime physics had read Galileo's works. That would have played the role of Einstein's "happiest thought" because Galileo came close to the conclusion that a falling body does not resist its fall:[64]

> But if you tie the hemp to the stone and allow them to fall freely from some height, do you believe that the hemp will press down upon the stone and thus accelerate its motion or do you think the motion will be retarded by a partial upward pressure? One always feels the pressure upon his shoulders when he prevents the motion of a load resting upon him; but if one descends just as rapidly as the load would fall how can it gravitate or press upon him? Do you not see that this would be the same as trying to strike a man with a lance when he is running away from you with a speed which is equal to, or even greater, than that with which you are following him? You must therefore conclude that, during free and natural fall, the small stone does not press upon the larger and consequently does not

[64] Galileo, *Dialogues Concerning Two Sciences*. In: S. Hawking (ed.), *On The Shoulders Of Giants*, (Running Press, Philadelphia 2002) pp. 399-626, p. 447

increase its weight as it does when at rest.

Then the path to the idea that gravitational phenomena are manifestations of the curvature of spacetime would have been open – the experimental fact that a falling particle accelerates (which means that its worldtube is curved), but offers no resistance to its acceleration (which means that its worldtube is not deformed) can be explained only if the worldtube of a falling particle is *both curved and not deformed*, which is impossible in the flat Minkowski spacetime where a curved worldtube is always deformed. Such a worldtube can exist only in a non-Euclidean spacetime whose geodesics are naturally curved due to the spacetime curvature, but are not deformed.

Second, imagine that after his *Space and Time* lecture Minkowski found a very challenging mathematical problem and did not compete with Einstein for the creation of the modern theory of gravitation. But when Einstein linked gravitation with the geometry of spacetime Minkowski regretted his change of research interests and started to study intensely general relativity and its implications.

As a mathematician he would be appalled by what he saw as confusing of physics and geometry:

- The new theory of gravitation demonstrates that gravitational physics is in fact geometry of curved spacetime; no general relativity of anything can be found there.

- How could physicists say that in the framework of general relativity *itself* gravitational phenomena are caused by gravitational *interaction*? According to what general relativity itself tells us *gravity is not a physical interaction* since by the geodesic hypothesis (confirmed by experiment) particles falling towards a planet and planets orbiting the Sun *all move by inertia* and inertial motion by its very nature presupposes *no interaction*. The mass of the Sun, for example, curves spacetime *no matter whether or not there are other planets in its vicinity*, and the planets move by inertia while orbiting the Sun (the correct expression is: the planets' worldlines are geodesics which represent inertial motion).

- How could physicists talk about gravitational energy in the framework of general relativity? There is no gravitational field and no gravitational force[65] (therefore there is no gravitational energy either since such energy is defined as the work done by gravitational forces); the gravitational field is at best a *geometric* not a physical field, and as such it does not possess any energy. Moreover, the mathematical formalism of general relativity *itself* refuses to yield a proper (tensorial) expression for gravitational energy and momentum.

I guess some physicists might be tempted to declare that such questions are obvious nonsense. For instance, they might say that the decrease of the orbital period of a binary pulsar system, notably the system PSR 1913+16 discovered by Hulse and Taylor in 1974, provided indirect experimental evidence for the existence of gravitational energy that is carried away by gravitational waves emitted by the neutrons stars in the system.

[65] What had been traditionally called gravitational force – the weight of a particle – turned out to be inertial force in spacetime physics. The worldline of a falling particle is geodesic (which means it moves by inertia), but when the particle touches the ground, it is prevented from moving by inertia and it resists the change of its state of inertial motion by exerting an inertal force on the ground. For more details see V. Petkov, "Physics as Spacetime Geometry" in A. Ashtekar, V. Petkov (eds), *Springer Handbook of Spacetime* (Springer, Heidelberg 2014), pp. 141-163, p. 150.

However, it should be reminded that such a statement constitutes a double contradiction with general relativity: the assertion that bodies, whose worldlines are geodesic, emit (i) gravitational waves and (ii) gravitational energy contradicts particularly the geodesic hypothesis and the experimental evidence which confirmed it.[66]

The neutron stars in the PSR 1913+16 system had been modeled by Taylor and Hulse "as a pair of orbiting point masses,"[67] which means that they are exact geodesics[68] and by the geodesic hypothesis bodies, whose worldlines are geodesics, *move by inertia*. But motion by inertia

- is motion without losing energy (since the very essence of inertial motion is motion without any loss of energy) and
- does not generate gravitational waves in general relativity.

Regarding the recently detected gravitational waves coming from colliding black holes – no gravitational waves are emitted when the black holes orbit each other before they collide (as the black holes are modelled as point masses, they are geodesic worldlines and no gravitational waves are generated by geodesic worldlines); when the black holes collide their worldlines are no longer geodesic and

[66] See a critical examination of the "confusing of physics and geometry" (as Minkowski might have called it) by explicitly following Minkowski's approach in V. Petkov, *Inertia and Gravitation: From Aristotle's Natural Motion to Geodesic Worldlines in Curved Spacetime* (Minkowski Institute Press, Montreal 2012), Appendix D.

[67] R.A. Hulse, J.H. Taylor, "Discovery of a pulsar in a binary system," *Astrophys. J.* **195** (1975) L51-L53.

[68] If the stars are regarded as extended bodies, then gravitational waves are emitted; see:
https://doingphysicsright.wordpress.com/2016/02/14/do-gravitational-waves-carry-gravitational-energy-and-momentum/.

gravitational waves are emitted,[69] but no gravitational energy is carried by the gravitational waves because of the reasons given above (as there exists no gravitational force, there is no gravitational energy either since such energy is defined as the work done by gravitational forces).

5. Minkowski and the reality of spacetime

Since 1908 there has been no consensus on the reality of the absolute four-dimensional world no matter whether it is the flat Minkowski spacetime or a curved spacetime since both spacetimes represent a *four-dimensional* world with time *wholly* given as the fourth dimension. What makes this issue truly unique in the history of science is that for over a hundred years not only has it remained an unresolved one, but for some it has been even a non-issue, whereas Minkowski had already provided the necessary evidence for the reality of spacetime in 1907 and 1908. He had fully realized the profound physical meaning of the relativity principle (reflecting the existing at his time experimental evidence) – *the impossibility to discover absolute motion experimentally unequivocally implies that observers in relative motion have different times and spaces, which in turn implies that what exists is an absolute four-dimensional world*.

Apparently Minkowski had realized the entire depth and grandness of the new view of the absolute four-dimensional world imposed on us by the experimental evidence. A draft of his Cologne lecture *Space and Time* reveals that he appears to have tried to tone down his excitement in the announcement

[69]From the official LIGO GW170104 Press Release (1 June 2017): "As was the case with the first two detections, the waves were generated when two black holes collided to form a larger black hole" (https://www.ligo.caltech.edu/page/press-release-gw170104).

of the unseen revolution in our understanding of the world. As the draft shows Minkowski's initial intention had been to describe the impact of the new world view in more detail – he had written that the essence of the new views of space and time "is mightily revolutionary, to such an extent that when they are completely accepted, as I expect they will be, it will be disdained to still speak about the ways in which we have tried to understand space and time."[70] In the final version of the lecture Minkowski had reduced this sentence about the new views of space and time to just "Their tendency is radical."

Given this rather restrained (compared to the draft version) announcement of the successful decoding of the physical meaning of the relativity principle – that the world is four-dimensional – it is surprising that Damour referred to that announcement as "the somewhat theatrical tone of Cologne's non-technical exposé."[71] The tone of the Cologne lecture could look theatrical only to someone who does not see the major issue in it in the way Minkowski saw it. This seems to be precisely the case since Damour apparently regards Minkowski's unification of space and time into an absolute four-dimensional world as nothing more than a mathematical abstraction:[72]

> Though Minkowski certainly went much farther than Poincaré in taking seriously the 4-dimensional geometry as a new basis for a physico-mathematical representation of reality, it does not seem that he went, philosophically and existentially, as far as really considering 'the flow

[70]See: P. L. Galison, Minkowski's Space-Time: From Visual Thinking to the Absolute World, *Historical Studies in the Physical Sciences*, **10** (1979) pp. 85-121, p. 98.

[71]T. Damour, "What is missing from Minkowski's "Raum und Zeit" lecture", *Annalen der Physik.* **17**, No. 9-10, (2008) pp. 619-630, p. 620.

[72]T. Damour, loc. cit., p. 626.

> of time' as an illusory shadow. By contrast, let us recall that the old Einstein apparently did take seriously, at the existential level, the idea that 'time' was an illusory shadow, and that the essence of (experienced) reality was timeless.

Minkowski's paper does not contain anything that even resembles a hint of what Damour wrote – that "it does not seem that he went, philosophically and existentially, as far as really considering 'the flow of time' as an illusory shadow."[73] On the contrary, the whole paper and even its "theatrical tone" (in Damour's own words) unambiguously demonstrates that Minkowski consciously announced a major discovery about the world, not a discovery of a mathematical abstraction (moreover Minkowski was fully aware that that mathematical abstraction was already published by Poincaré two years before Minkowski's Cologne lecture).

It is particularly disturbing when especially experts in spacetime physics do not regard spacetime as representing a *real* four-dimensional world and still hold the *unscientific*[74]

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[73] It is an obvious question: "How does Damour know that the flow of time is *not* an "illusory shadow"?" He does not have a hint of experimental or scientific evidence that supports the reality of the flow of time; see the next footnote. Damour's written defence of time flow is especially worring since he is an expert in spacetime physics, not a layman whose sometimes irrational and stubborn reaction to the implication of spacetime physics that time does not flow objectively – *this cannot be because it cannot be* – is neither science nor common sense.

[74] This everyday view is unscientific on two counts: (i) there is *no scientific evidence* whatsoever for the *sole* existence of the present moment, which is the central element of the concept of time flow (what is sufficient for the issue of the reality of spacetime is that there is no physical evidence for the existence of time flow). If the flow of time were a feature of the physical world (not of the image of the world in our mind), physics would have discovered it by now; (ii) the experimental evidence which confirmed the kinematic relativistic effects would be impossible if the world were *not* four-dimensional (see below), which means that the flow of time is indeed an illusion as Einstein believed.

view that time flows. Such an opinion of spacetime as nothing more than a mathematical space was openly defended by another physicist, Mermin, in a recent article *What's bad about this habit* in the May 2009 issue of *Physics Today* where he argued that "It is a bad habit of physicists to take their most successful abstractions to be real properties of our world." [75] He gave the issue of the reality of spacetime as an example – "spacetime is an abstract four-dimensional mathematical continuum" – and pointed out that it is "a bad habit to reify the spacetime continuum". Mermin specifically stressed that spacetime does not represent a real four-dimensional world: "The device of spacetime has been so powerful that we often reify that abstract bookkeeping structure, saying that we inhabit a world that is such a four- (or, for some of us, ten-) dimensional continuum."

I think the proper understanding of Minkowski's spacetime *physics* (which requires more effort than learning its four-dimensional formalism) is crucial not only for deep understanding of modern physics, but more importantly such understanding is a necessary condition for making discoveries in the twenty-first century physics.

The best proof that the *experimental evidence* against the existence of absolute motion (reflected in the relativity postulate) implies that the Universe is an absolute four-dimensional world is contained in Minkowski's paper itself. As discussed in Section 2 Minkowski first realized the important hidden message in the *experimental fact* that physical phenomena are the same in all inertial reference frames (which Einstein merely stated in the relativity postulate without *explaining* it) – physical phenomena are the same in all inertial reference frames because every inertial observer has his own space and time[76] and therefore describes

[75] N. D. Mermin, What's Bad About This Habit? *Physics Today* 2009, p. 8.

[76] As we saw in Section 2 Minkowski showed that the equivalence of

the phenomena in his reference frame (i.e. in his own space and time) in which he is at rest. For example, the Earth is at rest with respect to its space and therefore all experiments confirm this state of rest. Due to his excellent geometrical imagination Minkowski appears to have immediately realized that many spaces are possible in a four-dimensional world. In this way he managed to decode the physical meaning of the experimental fact that absolute motion cannot be discovered – that fact implies that the Universe is an absolute four-dimensional world in which space and time are inseparably amalgamated; *only in such a world one can talk about many spaces and many times.* Minkowski noted that "I think the word *relativity postulate* used for the requirement of invariance under the group G_c is very feeble. Since the meaning of the postulate is that through the phenomena only the four-dimensional world in space and time is given, but the projection in space and in time can still be made with certain freedom, I want to give this affirmation rather the name *the postulate of the absolute world*" (this volume).

To see why Minkowski's absolute four-dimensional world adequately represents the *dimensionality* of the real world, assume the opposite – that the real world is three-dimensional and time really flows (as our everyday experience so convincingly appears to suggest). Then there would exist just *one* space, which as such would be *absolute* (i.e. it would be the *same* for all observers since only a *single* space would exist). This would imply that absolute motion should exist and therefore there would be no relativity principle.

Another example of why special relativity (as we now call the physics of flat spacetime) would be *impossible in a three-dimensional world* is contained in Minkowski's four-dimensional explanation of the physical meaning of length

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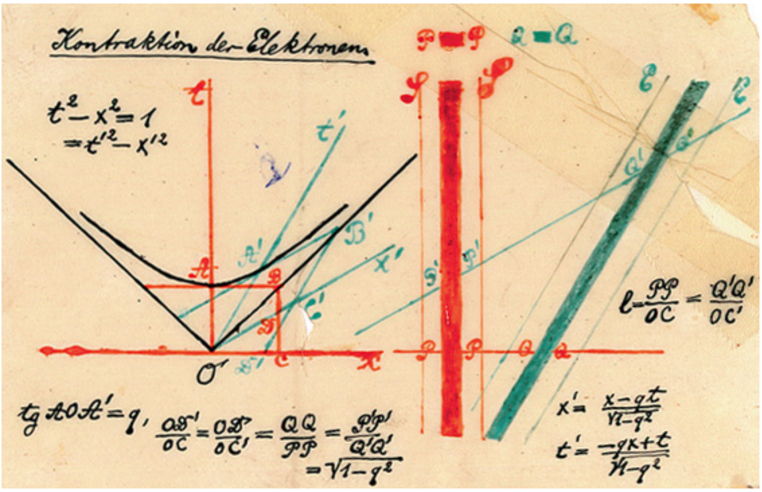
the times of observers in relative motion (which is necessary to explain why absolute motion cannot be detected) means that the observers have not only different times but different spaces as well.



The transparency which Minkowski used at his lecture in Cologne on September 21, 1908. It shows Fig. 1 in his paper (this volume). Source: Cover of *The Mathematical Intelligencer*, Volume 31, Number 2 (2009).

contraction, which is shown in the above figure (displaying the transparency Minkowski used in 1908). Consider only the vertical (red) strip which represents a body at rest with respect to an observer. The proper length of the body is the cross section PP of the observer's space, represented by the horizontal (red) line, and the body's strip. The relativistically contracted length of the body measured by an observer in relative motion with respect to the body is the cross section $P'P'$ of the moving observer's space, represented by the inclined (green) line, and the body's strip (on the transparency $P'P'$ appears longer than PP because the two-dimensional pseudo-Euclidean spacetime is represented on the two-dimensional Euclidean surface of the page).

To see that no length contraction would be possible in a three-dimensional world,[77] assume that the world

[77]A visual representation of Minkowski's explanation of length contraction is given in V. Petkov, *Spacetime and Reality: Facing the Ultimate Judge*, Sect. 3 (http://philsci-archive.pitt.edu/9181/).

is indeed three-dimensional. This would mean that all objects are also three-dimensional. Therefore the four-dimensional vertical strip of the body would not represent anything real in the world and would be merely an abstract geometrical construction. Then, obviously, the cross sections PP and $P'P'$ would coincide and there would be no length contraction since the observers in relative motion would measure the *same* three-dimensional body which has just *one* length $PP \equiv P'P'$.

The impossibility of length contraction in a three-dimensional world also follows even without looking at the spacetime diagram: it follows from the definition of a three-dimensional body – all its parts which exist *simultaneously* at a given moment; when the two observers in relative motion measure the length of the body, they measure *two different three-dimensional bodies* since the observers have *different* sets of simultaneous events, i.e. different sets of simultaneously existing parts of the body (which means two different three-dimensional bodies). If the world and the physical bodies were three-dimensional, then the observers in relative motion would measure the *same* three-dimensional body (i.e. the same set of simultaneously existing parts of the body), which means that (i) they would have a *common set of simultaneous events* in contradiction with relativity (simultaneity would be absolute), and (ii) they would measure the *same* length of the body, again in contradiction with relativity.

The same line of reasoning demonstrates that no relativity of simultaneity, no time dilation, and no twin paradox effect would be possible in a three-dimensional world.[78]

As I gave examples of how some physicists do not fully

I think this representation most convincingly demonstrates that length contraction is impossible in a three-dimensional world.

[78] V. Petkov, *Relativity and the Nature of Spacetime*, 2nd ed. (Springer, Heidelberg 2009) Chap. 5.

appreciate the depth of Minkowski's discovery that the physical world is four-dimensional, it will be fair to stress that there have been many physicists (I would like to think the majority) who have demonstrated in written form their brilliant understanding of what the dimensionality of the world is. Here are several examples.

A. Einstein, *Relativity: The Special and General Theory* (Routledge, London 2001) p. 152:

> It appears therefore more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the *evolution* of a three-dimensional existence.

A. S. Eddington, *Space, Time and Gravitation: An Outline of the General Relativity Theory* (Cambridge University Press, Cambridge 1920), p. 51:

> In a perfectly determinate scheme the past and future may be regarded as lying mapped out – as much available to present exploration as the distant parts of space. Events do not happen; they are just there, and we come across them.

A. S. Eddington, *Space, Time and Gravitation: An Outline of the General Relativity Theory* (Cambridge University Press, Cambridge 1920), p. 56:

> However successful the theory of a four-dimensional world may be, it is difficult to ignore a voice inside us which whispers: "At the back of your mind, you know that a fourth dimension is all nonsense." I fancy that that voice must often have had a busy time in the past history of physics. What nonsense to say that this solid table on which I am writing is a collection of

> electrons moving with prodigious speeds in empty spaces, which relatively to electronic dimensions are as wide as the spaces between the planets in the solar system! What nonsense to say that the thin air is trying to crush my body with a load of 14 lbs to the square inch! What nonsense that the star cluster which I see through the telescope obviously there now, is a glimpse into a past age 50 000 years ago! Let us not be beguiled by this voice. It is discredited.

Eddington made his most explicit comment on the reality of spacetime when he discussed the fact (discovered by Minkowski) that not only do observers in relative motion have different times but they also have different spaces, which however are fictitious since according to Minkowski the four-dimensional world is not objectively divided into such spaces and times (A.S. Eddington, The Relativity of Time, *Nature* **106** (1921) pp. 802–804, p. 803):

> It was shown by Minkowski that all these fictitious spaces and times can be united in a single continuum of four dimensions. The question is often raised whether this four-dimensional space-time is real, or merely a mathematical construction; perhaps it is sufficient to reply that it can at any rate not be less real than the fictitious space and time which it supplants.

H. Weyl, *Philosophy of Mathematics and Natural Science* (Princeton University Press, Princeton 2009) p. 116:

> The objective world simply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting

image in space which continuously changes in time.

H. Weyl, *Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics* (Princeton University Press, Princeton 2009) p. 135:

> The objective world merely exists, it does not happen; as a whole it has no history. Only before the eye of the consciousness climbing up in the world line of my body, a section of this world "comes to life" and moves past it as a spatial image engaged in temporal transformation.

R. Geroch, *General Relativity: 1972 Lecture Notes* (Minkowski Institute Press, Montreal 2013), p. 7:

> There is no dynamics in spacetime: nothing ever happens there. Spacetime is an unchanging, once-and-for-all picture encompassing past, present, and future.

In a real four-dimensional world there is no time flow since *all moments of time have equal existence* as they all form the fourth dimension (which like the other three dimensions is *entirely given*), whereas the very essence of time flow is that *only one moment of time exists* which constantly changes. But it is a well known fact that there does not exist any physical evidence whatsoever that only the present moment exists. On the contrary, the relativistic experimental evidence confirms Minkowski's view that all moments of time have equal existence due to their belonging to the entirely given time dimension. So the "old" Einstein was wise[79] to take

[79] I think it is this context that is the right and fair one for using the word 'old' especially if it refers to such a scientist and a person as Einstein.

seriously the absolute four-dimensional world and the idea that the flow of time was merely "a stubbornly persistent illusion" as evident from his letter of condolences to the widow of his longtime friend Besso:[80]

> Now Besso has departed from this strange world a little ahead of me. That means nothing. People like us, who believe in physics, know that the distinction between past, present and future is only a stubbornly persistent illusion.

Minkowski succeeded in demonstrating how the power of mathematical thinking applied to unresolved physical problems can free us from such illusions and can reveal the existence of a reality that is difficult to comprehend at once. Galison masterfully summarized the essence of Minkowski's discovery by pointing out that in his lectures *The Relativity Principle* and *Space and Time* "the idea is the same: beyond the divisions of time and space which are imposed on our experience, there lies a higher reality, changeless, and independent of observer."[81]

I think there are still physicists and philosophers who have been effectively refusing to face the implications of a real four-dimensional world due to the huge challenges they pose. But trying to squeeze Nature into our pre-set and deceivingly comfortable views of the world should not be an option for anyone in the 21st century.

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[80] Quoted from: Michele Besso, From Wikipedia, the free encyclopedia (`http://en.wikipedia.org/wiki/Michele_Besso`). Besso left this world on 15 March 1955; Einstein followed him on 18 April 1955.

[81] P. L. Galison, Minkowski's Space-Time: From Visual Thinking to the Absolute World, *Historical Studies in the Physical Sciences*, **10** (1979) pp. 85-121, p. 98.