

Lorentz Transformations and Existence in Minkowski Spacetime

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Overview of Talk

This talk will

- I. Introduce a novel interpretation of Lorentz contraction and time dilation.
- II. Use this reinterpretation to bring attention to four unappreciated spacetime principles, from which an *ontic equivalence relation* is derived.
- III. Touch on some interesting implications of the re-interpretation and the ontic equivalence relation.

An Informal Preview of the Re-interpretation

- 1 **Dimensional abatement:** As an object is length-contracted it attains an greater two-dimensional character, up until in the limit of c , when the contraction is complete and the object is dimensionally reduced.
- 2 **Ontochronic abatement:** As an object is time-dilated, its duration of existence in spacetime between two given spacetime events is diminished, up until in the limit of c , when time dilation is complete and its duration of existence in spacetime between spacetime events is exactly zero.
- 3 **Note:** The re-interpretation is of course *in addition to*, rather than instead of, the standard interpretation.

Lorentz Contraction as Dimensional Abatement I

Definition

Absolute Dimensionality: *The absolute dimensionality of an object is a dimensionless natural number that refers to the independent length dimensions which characterize it.*

Definition

Volume-Boundary ratio: *The Volume-Boundary ratio of a compact object with absolute dimensionality $n > 1$ is the ratio of its n -dimensional volume to its $n - 1$ -dimensional boundary.*

Definition

Relative Dimensionality: *Relative Dimensionality is the dimensionless ratio of the Volume-Boundary ratio of a compact object with absolute dimensionality $n > 1$ to that of a compact reference object, also with absolute dimensionality n .*

Lorentz Contraction as Dimensional Abatement II

$$\dim_{\text{rel}}(a/b) = \frac{\frac{\int dV_a}{\int dA_a}}{\frac{\int dV_b}{\int dA_b}} \text{ where}$$

- a is the comparison object, $\int dV_a$ its volume, $\int dA_a$ its surface area
- b is the reference object, $\int dV_b$ its volume, $\int dA_b$ its surface area
- $\dim_{\text{rel}}(a/b)$ is the relative dimensionality of a to b in three space dimensions,.

Note: $\dim_{\text{rel}}(a/b)$ is a dimensionless measure of the “dimensional character” of a relative to b , but when a and b have identical shape, then it also becomes a measure of the size of a relative to b .

Definition

Dimensional Diminution: For an n -dimensional compact object, dimensional diminution is the decrease of its relative dimensionality compared to its original state to a real number in the open interval $(0, 1)$.

Definition

Dimensional Reduction: For an n -dimensional compact object ($n > 1$), dimensional reduction is the decrease of its absolute dimensionality to $n - 1$. Equivalently, it is the decrease of its relative dimensionality compared to its original state to 0.

Lorentz Contraction as Dimensional Abatement III

Definition

Dimensional Abatement: *A less specific umbrella term which can either refer to Dimensional Diminution or to Dimensional Reduction.*

Proposition

Lorentz contraction can be conceptualized in terms of dimensional abatement. More specifically, it signifies dimensional diminution for $0 < v < c$ and dimensional reduction for $v = c$.

Proof: Consider a compact body B moving in a frame S and a moving frame S' in which B is at rest. We imagine B in S' as being made out of infinitesimal cubical volume elements oriented, without loss of generality, such that the direction of contraction in S will be normal to one of the sides. It is trivial to show that the Lorentz contraction of each cubical element in S causes it to be dimensionally abated. Since this is true of every infinitesimal volume element of B , it is true of B . ■

A Criterion for Physical Existence in Spacetime

Arguably, our understanding of nature has become so deep that in order to make further progress, we need to incorporate the concept of existence into physics. The following existence criterion, presented as an axiom, is an attempt to do so:

Criterion

A physical object exists in Minkowski spacetime if and only if it is characterized by a timelike spacetime interval.

Time Dilation as Ontochronic Abatement I

Definition

Spacetime Ontic Function: *The spacetime ontic function is a map $\exists_S : \mathfrak{D} \rightarrow \{0, 1\}$ where \mathfrak{D} is the set of all physical objects taken to be within the domain of physics and $S \subset \mathfrak{D}$ is the subset of \mathfrak{D} of all objects that exist in spacetime. The spacetime ontic value of an object is determined by whether it satisfies the existence criterion ($\exists_S(x) = 1$) or not ($\exists_S(x) = 0$).*

Definition

Ontochronicity: *Ontochronicity is the quality of having a duration of physical existence.*

Definition

Relative Ontochronicity: *Relative ontochronicity is the dimensionless ratio of the the observed duration of existence of an object compared to that of a reference object, usually the observer.*

Time Dilation as Ontochronic Abatement II

$$\text{ont}_{\text{rel}}(a/b) = \frac{\int d\tau_a}{\int d\tau_b} \text{ where}$$

- a is the comparison object and $\int d\tau_a$ will turn out to be its proper time
- b is the reference object and $\int d\tau_b$ will turn out to be coordinate time.
- $\text{ont}_{\text{rel}}(a/b)$ is the relative ontochronicity of a to b

Note: When b is an observer observing a , we can write $\int d\tau_a = \tau$, $\int d\tau_b = t$ and thus $\text{ont}_{\text{rel}}(a/b) = \frac{\tau}{t}$ which is similar to, but distinct from $\gamma^{-1} = \frac{d\tau}{dt}$. In situations in which the context is clear, the definition may be relaxed to subsume γ^{-1} .

Definition

Ontochronic Diminution: *Ontochronic diminution is the decrease of the observed duration of existence of an object in a given time interval by a dimensionless factor in the open interval (0, 1).*

Definition

Ontic Reduction: *Ontic reduction is the reduction of the ontic value of an object to 0.*

Time Dilation as Ontochronic Abatement III

Definition

Ontochronic Abatement: *Ontochronic abatement is a less specific umbrella term which can either refer to ontochronic diminution or to ontic reduction.*

Proposition

Relativistic time dilation can be conceptualized in terms of ontochronic abatement. More specifically, it signifies ontochronic diminution for $0 < v < c$ and ontic reduction for $v = c$.

Proof: Follows trivially from re-interpreting the proper time of an object as its observed duration of existence in spacetime, and coordinate time as the duration of existence in spacetime of the observer, between two given spacetime events. ■

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Four Unappreciated Spacetime Principles

The reinterpretation focuses attention on two invariance and two symmetry principles:

- 1 **Invariance of Absolute Dimensionality:** *The absolute dimensionality of any compact body is invariant under spacetime coordinate transformations.*
- 2 **Homodimensionality of Space:** *The dimensionality of every (maximally dimensional) space-like hypersurface of Minkowski spacetime is everywhere the same.*
- 3 **Invariance of Spacetime Ontic Value:** *The spacetime ontic value of any compact body is invariant under spacetime coordinate transformations.*
- 4 **Homodimensionality of Time:** *The dimensionality of every timelike hypersurface of Minkowski spacetime is everywhere the same.*

The Relationship Between the Spacetime Principles

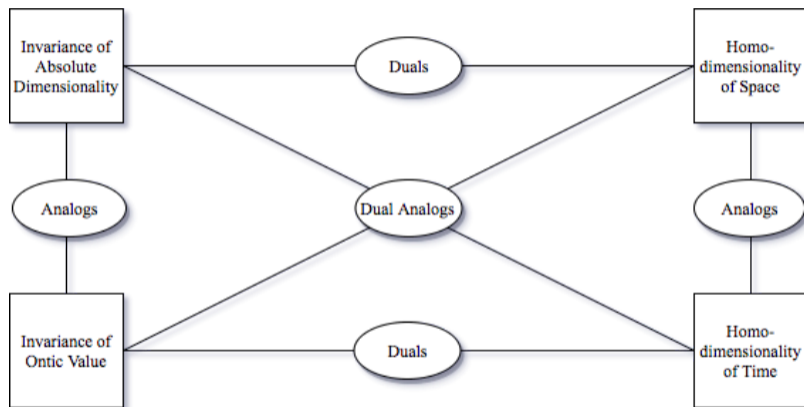


Figure: A diagram of the relationships between the spacetime principles

The Relationship Between the Spacetime Principles and the LT

- 1 The Lorentz transformations ensure that an object observed to be 3-dimensional in one spacetime frame will never be observed to be 2-dimensional in any other spacetime frame, and vice versa.
 - Lorentz contraction obeys the invariance of absolute dimensionality.
- 2 The Lorentz transformations ensure that an object observed to have a finite duration of existence in spacetime in one spacetime frame will never be observed to have a zero duration of existence in spacetime in any other spacetime frame, and vice versa.
 - Time Dilation obeys invariance of spacetime ontic value

Existence as an Equivalence Relation I

- **Principles 1 and 3 together couple absolute dimensionality to spacetime ontic value.** Propositions 1 and 2 together already show that dimensional and ontochronic diminution couple to each other exactly as Lorentz contraction and time dilation couple to each other, but the two invariance principles together extend this to dimensional and ontic reduction.
- **Principles 2 and 4 together ensure that the coupling of absolute dimensionality to spacetime ontic value holds globally.** In a spacetime in which the homodimensionality of space or of time fails to hold, there could conceivably be regions in which spacelike or timelike hypersurfaces have a different dimensionality inside the region than outside, and in such regions absolute dimensionality and ontic value could decouple. The two homodimensionality principles together ensure that this does not happen.

Definition

Isodimensional: *A Spacetime is isodimensional if and only if both its timelike and maximally dimensional spacelike hypersurfaces are homodimensional.*

Existence in spacetime as an Equivalence Relation II

Proposition

Physical existence in Minkowski spacetime is an equivalence relation by absolute dimensionality.

Proof: An equivalence relation is determined by the properties of reflexivity, symmetry and transitivity. Consider an n -dimensional compact object A subject to the above principles. By the the coupling of ontic value to absolute dimensionality, it must exist in an $n + 1$ dimensional Minkowski spacetime region. By the isodimensionality of Minkowski spacetime, this region is, in fact, all of $n + 1$ dimensional spacetime. In particular, A exists in the $n + 1$ -dimensional Minkowski spacetime in which it exists. This proves reflexivity. Now consider an m -dimensional compact object B . By the same argument as given for reflexivity, it must exist in an $m + 1$ dimensional spacetime. Suppose A exists in the same spacetime as B . This requires that $n + 1 = m + 1$, and, consequently, that $n = m$. But that means B has the same absolute dimensionality as A , and therefore exists in the same spacetime as A . This proves symmetry. Finally, consider an l -dimensional compact object C . By the same argument as given for reflexivity, it must exist in an $l + 1$ -dimensional spacetime. Now suppose that B exists in the same spacetime as C , and that A exists in the same spacetime as B . This requires $m + 1 = l + 1$ and $n + 1 = m + 1$, respectively, from which it follows that $n = m = l$, so A has the same absolute dimensionality as C and therefore exists in the same spacetime as C . This proves transitivity. ■

A Partition on all Things that Physically Exist *per se*

The ontic equivalence relation considered here partitions the set of all objects that physically exist *per se* into *ontic equivalence classes* such that for each $n + 1$ dimensional Minkowski spacetime, there is a corresponding equivalence class of n -dimensional objects that exist in it.

0+1 Spacetime	1+1 Spacetime	2+1 Spacetime	3+1 Spacetime	4+1 Spacetime	...
0-dimensional objects	1-dimensional objects	2-dimensional objects	3-dimensional objects	4-dimensional objects	...

Figure: A partition of all physically existing objects into ontic equivalence classes by absolute dimensionality.

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Some Interesting Implications

- 1 Objects in Spacetime affect its topological structure.
- 2 Classical Electromagnetic fields can be reconceptualized in a more fundamental way
- 3 The ontic equivalence relation applies more generally than just to Minkowski
- 4 Theories which assume spacetimes with “extra” dimensions together with the four spacetime principles may be inconsistent.
- 5 Speed-of Light Objects belong to a different ontic equivalence class than spacetime observers
- 6 Ontic Value permits the separation of the quantum from the classical domain

1. Objects in Spacetime Affect its Topological Structure

AIP Journal of Mathematical Physics

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Journal of Mathematical Physics **18**, 1399 (1977); <https://doi.org/10.1063/1.523436>

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PDF ABSTRACT CITED BY TOOLS SHARE METRICS

TOPICS
• Classical general relativity

ABSTRACT
The title assertion is proven, and two corollaries are established. First, the topology of every past and future distinguishing spacetime is determined by its causal structure. Second, in every spacetime the path topology of Hawking, King, and McCarthy codes topological, differential, and conformal structure.

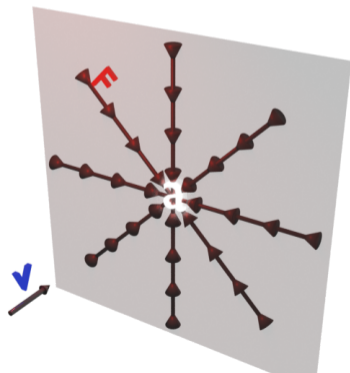
REFERENCES

Scilight

- 1 Malament proved that the class of continuous timelike curves determines the topology of spacetime.
- 2 Objects on continuous timelike curves satisfy the existence criterion.
- 3 A slogan: *In special relativity, objects in spacetime affect its topology; in general relativity, they also affect its geometry.*

2. Classical Electromagnetic fields can be reconceptualized in a more fundamental way

- Unlike length contraction, dimensional Abatement can be applied to infinitely extended fields
- Can think of the magnetic **Force** field of a point charge as the line integral of a 1-dimension reduced analog of its electric **Force** field and derive from it $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$



3. The Ontic Equivalence Relation Applies More Generally

- ① We tacitly assume the four spacetime principles apply to spacetimes other than Minkowski because spacetimes which violate them would seem sufficiently weird for us to notice the violation.
 - ① Ordinarily, we take it as part of the *definition* of a pseudo-Riemannian manifold (M, g) that it is isodimensional. If M is isodimensional, then the two homodimensionality principles hold in M .
 - ② The invariance of absolute dimensionality and spacetime ontic value seem to be assumed without an accompanying definition.
- ② If the homodimensionality and invariance principles hold in M , then the ontic equivalence relation also holds in M .
- ③ **But:** It appears that singularities (e.g. inside Black holes) denote point-like spatial regions in which the four spacetime principles may not hold.

Definition

Isodimensional almost everywhere: *A spacetime is isodimensional almost everywhere if and only if it is isodimensional up to a countable number of point-like (in space) singularities .*

Can modify proof of the ontic equivalence relation so that it holds 'almost everywhere' in M .

4. Theories which assume spacetimes with “Extra” Dimensions together with the Four Spacetime Principles *May be Inconsistent*

Here, “extra” dimensions refer to a situation in which an $m + 1$ -dimensional spacetime is taken to contain n -dimensional objects such that $m > n$.

- 1 If the n -dimensional object is taken to be able to freely locate anywhere in this spacetime, then the theory is inconsistent.
 - It follows from the principles that the matching of the absolute dimensionality of bodies with that of the space which contains them is not an “accident”.
- 2 However, there is a **loophole**: Impose a partition on the spacetime which confines the objects that exist in it within an $n + 1$ -dimensional subspace.
 - Modern speculative frameworks often assume this (KKT, Space-time-matter theory, String theory)
- 3 Classically, this removes the inconsistency
- 4 Quantum Theoretically, it is not not so clear because in QFT, distinction between particles and fields becomes blurry, and if a field does not obey the partition on the spacetime, the danger of inconsistency reappears (e.g. Randall-Sundrum Model). Ultimately, consistency needs to be determined by those who study these models in detail.

Are Unavailable Spatial Degrees of Freedom Real?

- 1 The Loophole also raises a metaphysical problem: In theories that postulate extra dimensions that belong to a separate equivalence class than those in which objects exist, **the unavailability of spatial degrees of freedom has now morphed from a heuristic demand to a mathematical necessity.**
- 2 That makes it imperative to find a *mathematical* explanation, ideally from established physics, for the partition on the spacetime: We can explain a heuristic demand by “this is how the physical world is”, but not a mathematical necessity.
 - E.g heuristic explanations like “the extra dimensions belong to a different equivalence class because they are compactified, and they are compactified because we do not observe them” are inadequate to explain a mathematical necessity because they treat it as though it were an “accident” of nature.
- 3 Absent a mathematical explanation, there is little to foreclose the possibility that these unavailable spatial degrees of freedom within the spacetime in which we exist are pure fantasy.

5. Speed-of-light objects belong to a different ontic equivalence class than spacetime observers

- This can now be given as an *explanation* for the impossibility of transforming to the rest frame of a speed-of-light object: If a spacetime observer could transform to a speed-of-light rest frame, he or she would no longer be a *spacetime* observer.
- We have been desensitized by a deluge of science fiction-y speculations in modern physics (Extra dimensions, multiverse, MUH, simulation hypothesis etc). Because of that, when a straightforward but paradigm-shifting implication of an established theory of physics is discovered, we may not appreciate its significance.
- The discovery that speed of light objects belong to a different ontic equivalence class than spacetime observers implies that **it is possible for something to exist without existing in spacetime**. Arguably, this is comparable to the discovery, some 500 years ago, that it is possible for something to be a sun without it being our own sun, namely a star.
- Yet, from its inception, special relativity, one of our best and most established scientific theories, held this implication right before us: The duration of existence of a photon in spacetime, from the moment it is emitted to the moment it is absorbed somewhere else, is exactly zero. We did not notice the significance of this because its implication does not fit into the prevailing worldview.

6. Ontic Value May Permit the Separation of the Quantum from the Classical Domain

- Photons are inherently quantum. If they belong to an equivalence class of objects distinct from the equivalence class of spacetime objects, then this suggests the possibility that all quantum objects belong to a distinct ontic equivalence class.
- A working hypothesis: Quantum systems fail to exist as a spacetime objects, but manifest as a combination of possibilities until spacetime objects emerge out of as yet unspecified interaction ⇒ **Measurement**
- Spacetime ontic value may be what keeps the domains of applicability of quantum and classical physics mutually exclusive.
- One potential problem: Quantum systems are often associated with mass, so need to introduce the concept of “mass as a potentiality”, or what I call *actualizable mass*¹. This is a falsifiable concept which has not been tested yet. Associate actualizable mass with a spacetime ontic value of 0 and let it characterize the mass of quantum systems.
- This is the main focus of my research program

¹ Are the Concepts of Mass in Quantum Mechanics and in General Relativity the same? AN
<https://deepblue.lib.umich.edu/handle/2027.42/87999>

Conclusion

- There was once before a re-interpretation of the Lorentz transformations, in 1905. It profoundly changed the course of 20th century physics and led to a *paradigm shift*.
- It is possible that the reinterpretation presented here may also lead to a paradigm shift. More work needs to be done to work out the consequences.
- The ideas presented here are discussed in papers available at *Deep Blue*, the University of Michigan's repository for scholarly work:
 - "Dimensionality in Physics": <http://hdl.handle.net/2027.42/147435>
 - "Existence in Physics": <http://hdl.handle.net/2027.42/147436>
- I welcome your questions, comments and criticisms, either in person or at armin@umich.edu

Thank you and have a safe trip home!