Perturbatively renormalizable quantum gravity

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Abstract

If one follows the standard procedures of perturbative quantum field theory, then one finds that quantum gravity suffers from the problem that it is not perturbatively renormalizable [1].

In gravity the natural coupling constant is $\kappa = 2/M$, where $\kappa^2 = 32\pi G/\hbar c$, and M is the reduced Planck mass. Given that κ has negative mass dimension, perturbative non-renormalizability is already expected from simple power counting arguments. We will show however that a genuine perturbative continuum limit for quantum gravity does exist, and that properties already inherent to the Einstein-Hilbert action tell us how it should be constructed. Although this theory is perturbative in κ, it is non-perturbative in Planck's constant, $\hbar$.

To understand why there is this possibility, one needs to work with the deeper understanding of renormalization afforded by the Wilsonian renormalization group (RG). For this one must work in Euclidean signature. This leads to the infamous "conformal factor instability". The Einstein-Hilbert action, S_{EH}, is the integral over the scalar curvature. Since there exist Euclidean manifolds with arbitrarily large curvature of either sign, the action cannot be bounded below.[1] It means that the functional integral from which we would hope to construct the theory, is more than usually ill-defined. Expanding around flat space as

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}\,, \qquad \text{where} \qquad H_{\mu\nu} = h_{\mu\nu} + \tfrac{1}{2}\,\varphi\,\delta_{\mu\nu}\,,$$

$h_{\mu\nu}$ being the traceless part, and φ the traceful (conformal factor) part, and decoupling the two parts with the Feynman – De Donder gauge, the problem is clearly visible already in the kinetic term for φ which has the wrong sign for convergence of the partition function:

$$\mathcal{L}^{\text{kinetic}}_{EH} = \frac{1}{2}\,(\partial_\lambda h_{\mu\nu})^2 - \frac{1}{2}\,(\partial_\lambda \varphi)^2\ . \tag{1}$$

Previously the problem has been dealt with by continuing the conformal factor functional integral along the imaginary axis: $\varphi \mapsto i\varphi$ [2]. However this wrong sign does not invalidate

[1] In fact it is large positive curvature that is the problem.

the continuum version of the Wilsonian RG, the so-called Exact RG [3, 4]. Therefore we keep the conformal factor instability and use the Exact RG instead to define the continuum limit. We find that the 'wrong sign' has a profound effect on the RG properties in the φ sector. In particular around the above Gaussian fixed point, eqn. (1), it results in a Hilbert space of renormalizable operators involving arbitrarily high powers of the gravitational fluctuations. These interactions are characterised by being exponentially suppressed for large field amplitude, perturbative in κ but non-perturbative in $\hbar$. By taking a limit to the boundary of this Hilbert space, diffeomorphism invariance is recovered in the continuum quantum field theory. Thus the so-called conformal factor instability is the key that allows the construction of a genuine continuum limit for quantum gravity [5–9].

References

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