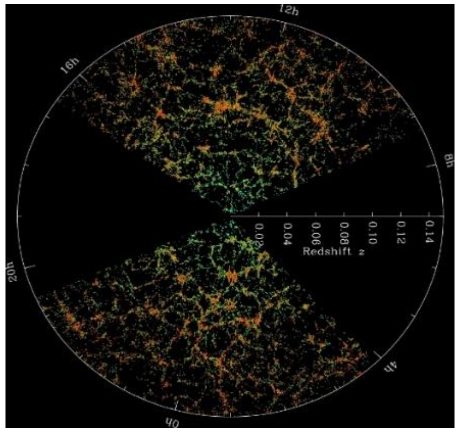
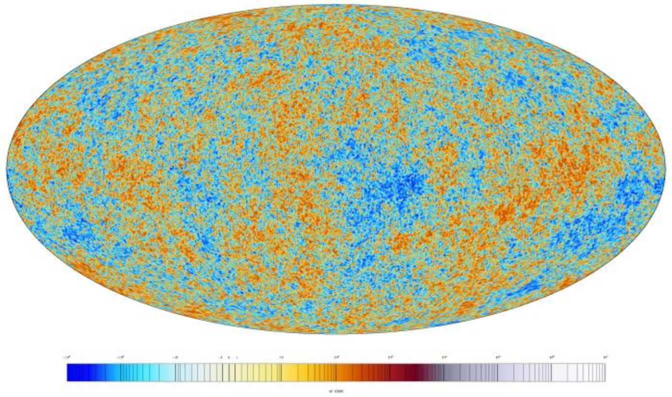
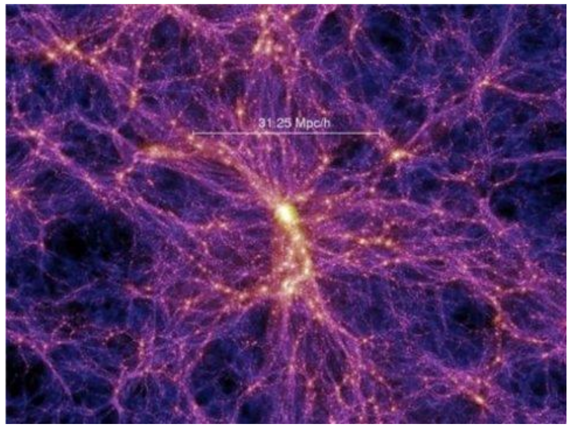
The effective fluid approach for Modify Gravity and Dark Energy

Rubén Arjona
IFT-UAM/CSIC







13 – 16 May 2019

Second Minkowski Meeting on the Foundation of Spacetime Physics

arXiv:1811.02469 R.Arjona, W.Cardona, S.Nesseris

arXiv:1904.06294 R.Arjona, W.Cardona, S.Nesseris

Summary

Modify gravity and **Dark energy** models are alternative scenarios for explaining the late-time acceleration of the Universe.

Provide simple **analytical formulae** for the equivalent dark energy effective fluid pressure, density and velocity for modify gravity and dark energy models.

Implement the dark energy effective fluid formulae in the **Boltzmann solver code** called CLASS.

Derive **constraints** from the latest cosmological data.

Main contents

- The Standard Cosmological Model
- The Effective Fluid Approach
- f(R) theories
- Horndeski theories
- Boltzmann solver codes: CLASS, hi_CLASS, EFCLASS
- Cosmological Constraints (MCMC)

The Standard Cosmological model (ΛCDM)

Five pillars

- General Relativity
- The Cosmological Principle
- The Cosmic Microwave Background
- The Big Bang Nucleosynthesis
- The Hubble law

Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Cosmological Constant

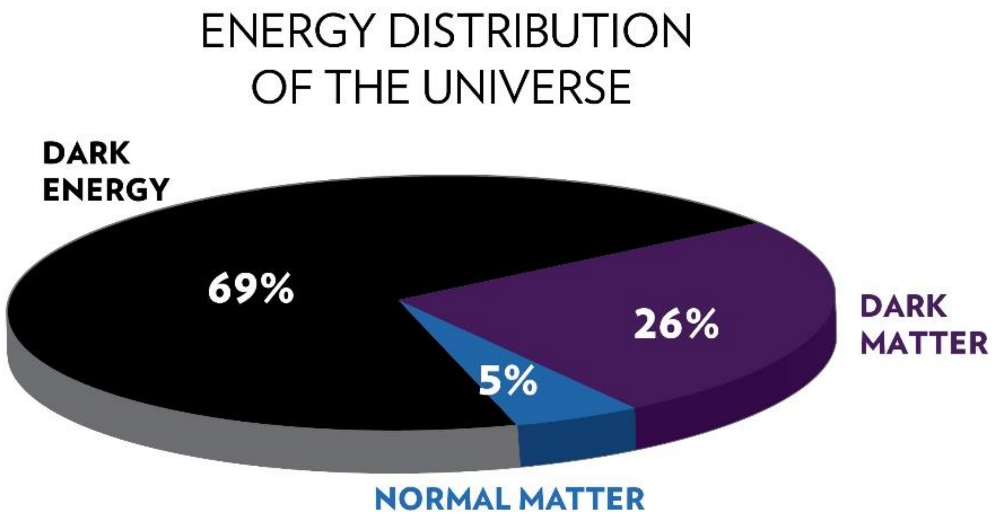
$$T^{\mu}_{\nu} = P g^{\mu}_{\nu} + (\rho + P) U^{\mu} U_{\nu}$$

Friedman - Lemaitre - Robertson - Walker (FLRW) metric

$$ds^2 = c^2 dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin(\theta)^2 d\phi^2 \right) \right)$$

The Standard Cosmological Model (ΛCDM)

The Universe is expanding…. but also **accelerating!**



$$G_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu} \qquad \kappa = \frac{8\pi G_N}{c^4}$$

ΛCDM simplest candidate

Fits most data sets. Good phenomenological model

Modified gravity theories

Why we need to go beyond GR ?

Small scales (UV)

Not renormalizable

(only at first loop)

Modified gravity theories can be String/quantum gravity inspired due to corrections expected in higher energies

Large scales (Infrared)

Dark energy

Add extra scalar field

Modified gravity

Quantum fields in curved space. Birrel and Davies

Explain the late-time acceleration of the Universe

2 leading approaches

Dark energy	***Modified gravity***
$G_{\mu\nu} = \kappa T_{\mu\nu} + (\cdots)$	$G_{\mu\nu} + (\cdots) = \kappa T_{\mu\nu}$
Keep GR introduce new fields and particles	Covariant modifications to GR

Effective Fluid Approach

Departures from GR can be interpreted as an effective fluid contribution

Variables describing the fluid

Background

$w(a)$ equation of state

Linear perturbations

$c_s^2(a,k)$ sound speed

$\pi(a,k)$ anisotropic stress

The linear order perturbations could in principle be distinguishable from the standard cosmological model

Theoretical framework

Perturbed FRW metric $ds^2 = a^2\left[-(1+2\Psi)d\tau^2 + (1-2\Phi)d\vec{x}^2\right]$ scalar

First order of perturbations

$$T^\mu_\nu = P g^\mu_\nu + (\rho + P)U^\mu U_\nu$$

$$\begin{cases} T^0_0 &= -(\bar\rho + \delta\rho) \\ T^0_i &= (\bar\rho + \bar P)u_i \\ T^i_j &= (\bar P + \delta P)\delta^i_j + \Sigma^i_j \end{cases}$$

Perturbed Einstein equations

$$k^2\Phi + 3\frac{\dot a}{a}\left(\dot\Phi + \frac{\dot a}{a}\Psi\right) = 4\pi G_N a^2 \delta T^0_0 \qquad (0,0)$$

$$k^2\left(\dot\Phi + \frac{\dot a}{a}\Psi\right) = 4\pi G_N a^2(\bar\rho + \bar P)\theta \qquad (0,i)$$

$$k^2(\Phi - \Psi) = 12\pi G_N a^2(\bar\rho + \bar P)\sigma \qquad (i,j)$$

Evolution equation for the perturbations

$\nabla_\nu T^{\mu\nu} = 0$ ⇨

$$\dot\delta = -(1+w)(\theta - 3\dot\Phi) - 3\frac{\dot a}{a}\left(c_s^2 - w\right)\delta \qquad \mu=0$$

$$\dot\theta = -\frac{\dot a}{a}(1-3w)\theta - \frac{\dot w}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\Psi \qquad \mu=i$$

arXiv:astro-ph/9506072v1

Theoretical framework

Evolution equation for the perturbations

$$\nabla_\nu T^{\mu\nu} = 0 \Rightarrow \begin{cases} \dot{\delta} = -(1+w)(\theta - 3\dot{\Phi}) - 3\frac{\dot{a}}{a}\left(c_s^2 - w\right)\delta & \mu=0 \\ \dot{\theta} = -\frac{\dot{a}}{a}(1-3w)\theta - \frac{\dot{w}}{1+w}\theta + \frac{c_s^2}{1+w}k^2\delta - k^2\sigma + k^2\Psi & \mu=i \end{cases}$$

Scalar velocity perturbation $V \equiv (1+w)\theta$

Anisotropic stress parameter $\pi = \frac{3}{2}(1+w)\sigma$

$$\nabla_\nu T^{\mu\nu} = 0 \Rightarrow \begin{cases} \delta' = 3(1+w)\Phi' - \frac{V}{a^2 H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right) \\ V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2 H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2 H}\Psi - \frac{2}{3}\frac{k^2}{a^2 H}\pi \end{cases}$$

Modified Gravity and Dark energy models

Effective fluid approach for specific models:

Toy model: f(R)

Horndeski theory

The effective fluid approach

$$S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + \int d^4x\mathcal{L}_M\left(g_{\mu\nu},\Psi_M\right)$$

Field equations

$$FG_{\mu\nu} - \frac{1}{2}\left(f(R) - RF\right)g_{\mu\nu} + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)F = \kappa T^{(m)}_{\mu\nu}$$

$$F = f'(R)$$

Eff. Fluid approach ⇒

$$G_{\mu\nu} = \kappa\left(T^{(m)}_{\mu\nu} + T^{(DE)}_{\mu\nu}\right)$$

$$\kappa T^{(DE)}_{\mu\nu} = (1-F)G_{\mu\nu} + \frac{1}{2}(f(R) - R\,F)g_{\mu\nu} - \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)F$$

$$\nabla^\mu T^{(DE)}_{\mu\nu} = 0$$

The effective fluid approach

$$\kappa T^{(DE)}_{\mu\nu} = (1-F)G_{\mu\nu} + \frac{1}{2}(f(R) - R\,F)g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F$$

Background Eqs.

$$\begin{cases} \mathcal{H}^2 = \frac{\kappa}{3}a^2\left(\bar{\rho}_m + \bar{\rho}_{DE}\right) \\ \dot{\mathcal{H}} = -\frac{\kappa}{6}a^2\left(\left(\bar{\rho}_m + 3\bar{P}_m\right) + \left(\bar{\rho}_{DE} + 3\bar{P}_{DE}\right)\right) \end{cases}$$

Effective DE density and pressure

$$\kappa\bar{P}_{DE} = \frac{f}{2} - \mathcal{H}^2/a^2 - 2F\mathcal{H}^2/a^2 + \mathcal{H}\dot{F}/a^2 - 2\dot{\mathcal{H}}/a^2 - F\dot{\mathcal{H}}/a^2 + \ddot{F}/a^2$$

$$\kappa\bar{\rho}_{DE} = -\frac{f}{2} + 3\mathcal{H}^2/a^2 - 3\mathcal{H}\dot{F}/a^2 + 3F\dot{\mathcal{H}}/a^2$$

DE equation of state

$$w_{DE} = \frac{-a^2 f + 2\left((1+2F)\mathcal{H}^2 - \mathcal{H}\dot{F} + (2+F)\dot{\mathcal{H}} - \ddot{F}\right)}{a^2 f - 6(\mathcal{H}^2 - \mathcal{H}\dot{F} + F\dot{\mathcal{H}})}$$

The effective fluid approach

$$\kappa T^{(DE)}_{\mu\nu} = (1-F)G_{\mu\nu} + \frac{1}{2}(f(R) - R\,F)g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F$$

Effective pressure, density and velocity perturbations

$$\frac{\delta P_{DE}}{\bar\rho_{DE}} = (...)\delta R + (...)\delta\dot R + (...)\delta\ddot R + (...)\Psi + (...)\dot\Psi + (...)\Phi + (...)\dot\Phi$$

$$\delta_{DE} = (...)\delta R + (...)\delta\dot R + (...)\Psi + (...)\Phi + (...)\dot\Phi$$

$$(1+w_{DE})\theta_{DE} = (...)\delta R + (...)\delta\dot R + (...)\Psi + (...)\Phi + (...)\dot\Phi$$

$$\Phi - \Psi = \frac{F_{,R}}{F}\delta R$$

In GR = 0!

$$\bar\rho_{DE}\pi_{DE} = \frac{1}{\kappa}\frac{k^2}{a^2}\left(F_{,R}\delta R + (1-F)(\Phi - \Psi)\right)$$

Sub-horizon approximation

Modes deep in the Hubble radius $k^2/a^2 \gg H^2$

Neglect time derivatives in the linearized Einstein equations

$$\delta R \simeq -\frac{4k^2}{a^2}\Phi + \frac{2k^2}{a^2}\Psi$$

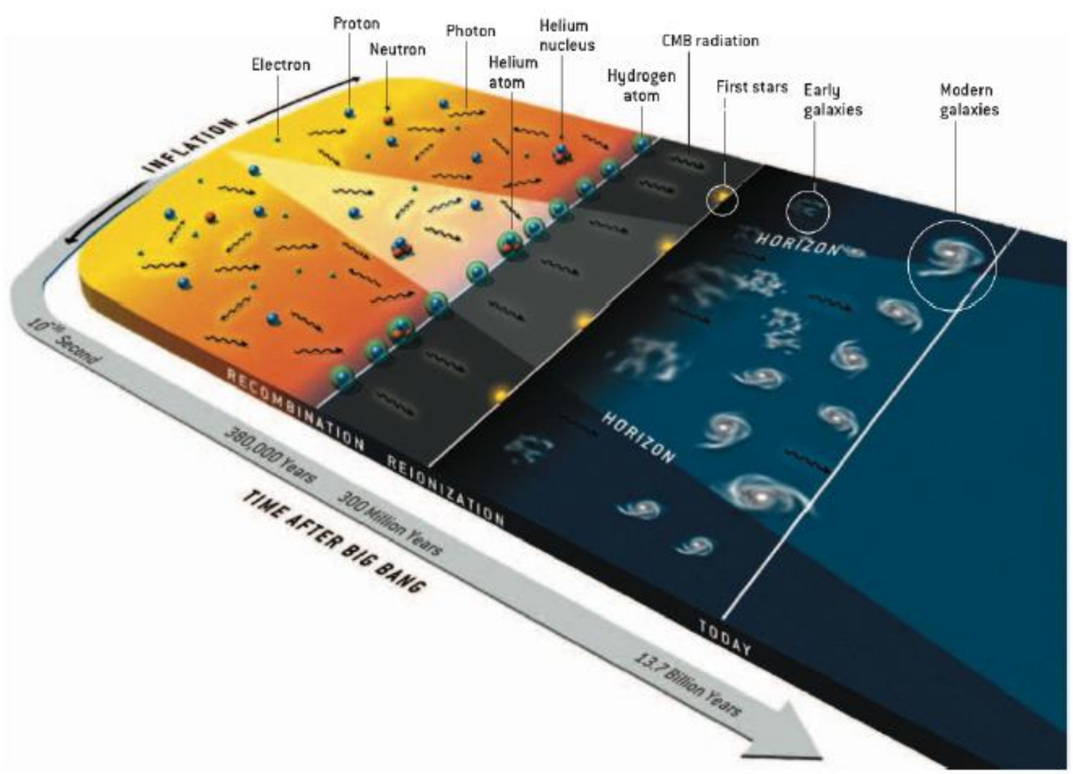
$$\Psi = -4\pi G_N \frac{a^2}{k^2}\frac{G_{eff}}{G_N}\bar{\rho}_m\delta_m$$

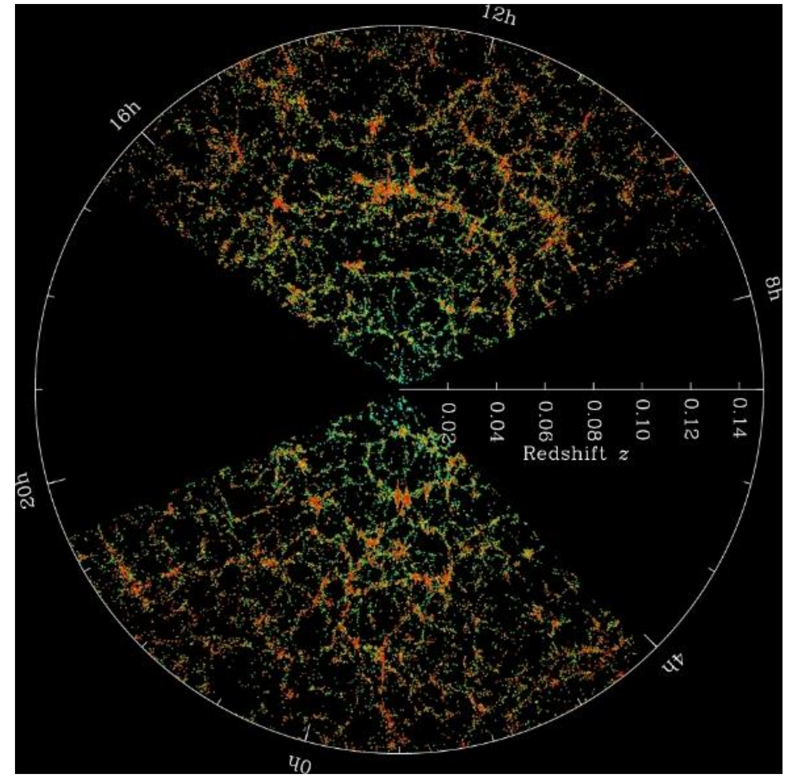
$$\Phi = -4\pi G_N \frac{a^2}{k^2} Q_{eff}\bar{\rho}_m\delta_m$$

$$G_{eff}/G_N = \frac{1}{F}\frac{1+4\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}$$

$$Q_{eff} = \frac{1}{F}\frac{1+2\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}$$

Growth of matter perturbations





We know that there are **matter** perturbations…but how do they grow?

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_N \rho_m \delta_m \simeq 0$$

G_{eff}

Growth of matter density perturbations on sub-horizon scales

$$\delta_m \equiv \frac{\delta\rho_m}{\rho_m}$$

perturbation

background

Measure how matter clusters

Sub-horizon approximation

Anisotropic parameters

$$\eta \equiv \frac{\Psi - \Phi}{\Phi} \text{ and } \gamma \equiv \frac{\Phi}{\Psi}$$

Departure from GR

$$\eta = \frac{2\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1 + 2\frac{k^2}{a^2}\frac{F_{,R}}{F}}$$

$$\gamma = \frac{1 + 2\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1 + 4\frac{k^2}{a^2}\frac{F_{,R}}{F}}$$

Sub-horizon approximation

$$\kappa T^{(DE)}_{\mu\nu} = (1-F)G_{\mu\nu} + \frac{1}{2}(f(R) - R\,F)g_{\mu\nu} - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\,F$$

Effective pressure, density and velocity perturbations

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} = (\ldots)\delta R + (\ldots)\delta\dot{R} + (\ldots)\delta\ddot{R} + (\ldots)\Psi + (\ldots)\dot{\Psi} + (\ldots)\Phi + (\ldots)\dot{\Phi}$$

$$\delta_{DE} = (\ldots)\delta R + (\ldots)\delta\dot{R} + (\ldots)\Psi + (\ldots)\Phi + (\ldots)\dot{\Phi}$$

$$(1+w_{DE})\theta_{DE} = (\ldots)\delta R + (\ldots)\delta\dot{R} + (\ldots)\Psi + (\ldots)\Phi + (\ldots)\dot{\Phi}$$

$$\bar{\rho}_{DE}\pi_{DE} = \frac{1}{\kappa}\frac{k^2}{a^2}\left(F_{,R}\delta R + (1-F)(\Phi - \Psi)\right)$$

Apply repeatedly

$$\Phi - \Psi = \frac{F_{,R}}{F}\delta R$$

Sub-horizon approximation

Effective pressure, density and velocity perturbations

$$\frac{\delta P_{DE}}{\bar{\rho}_{DE}} \simeq \frac{1}{3F}\frac{2\frac{k^2}{a^2}\frac{F_{,R}}{F} + 3(1+5\frac{k^2}{a^2}\frac{F_{,R}}{F})\ddot{F}k^{-2}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{1}{F}\frac{1-F+\frac{k^2}{a^2}(2-3F)\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$V_{DE} \equiv (1+w_{DE})\theta_{DE} \simeq \frac{\dot{F}}{2F}\frac{1+6\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$\pi_{DE} \simeq \frac{1}{F}\frac{\frac{k^2}{a^2}\frac{F_{,R}}{F}}{1+3\frac{k^2}{a^2}\frac{F_{,R}}{F}}\frac{\bar{\rho}_m}{\bar{\rho}_{DE}}\delta_m$$

$$c^2_{s,DE} \simeq \frac{1}{3}\frac{2\frac{k^2}{a^2}\frac{F_{,R}}{F} + 3(1+5\frac{k^2}{a^2}\frac{F_{,R}}{F})\ddot{F}k^{-2}}{1-F+\frac{k^2}{a^2}(2-3F)\frac{F_{,R}}{F}}$$

ΛCDM → 0

$$f(R) = R - 2\Lambda$$

The Hu & Sawicki (HS) model

$$f(R) = R - m^2 \frac{c_1 (R/m^2)^n}{1 + c_2 (R/m^2)^n}$$

c_1, c_2 are two free parameters, $m^2 \simeq \Omega_{m0} H_0^2$

After some algebraic manipulations

$$f(R) \; = \; R - \frac{2\Lambda}{1 + \left(\frac{b\Lambda}{R}\right)^n}$$

Small perturbation around ΛCDM

$$\lim_{b \to 0} f(R) \; = \; R - 2\Lambda$$

$$\lim_{b \to \infty} f(R) \; = \; R$$

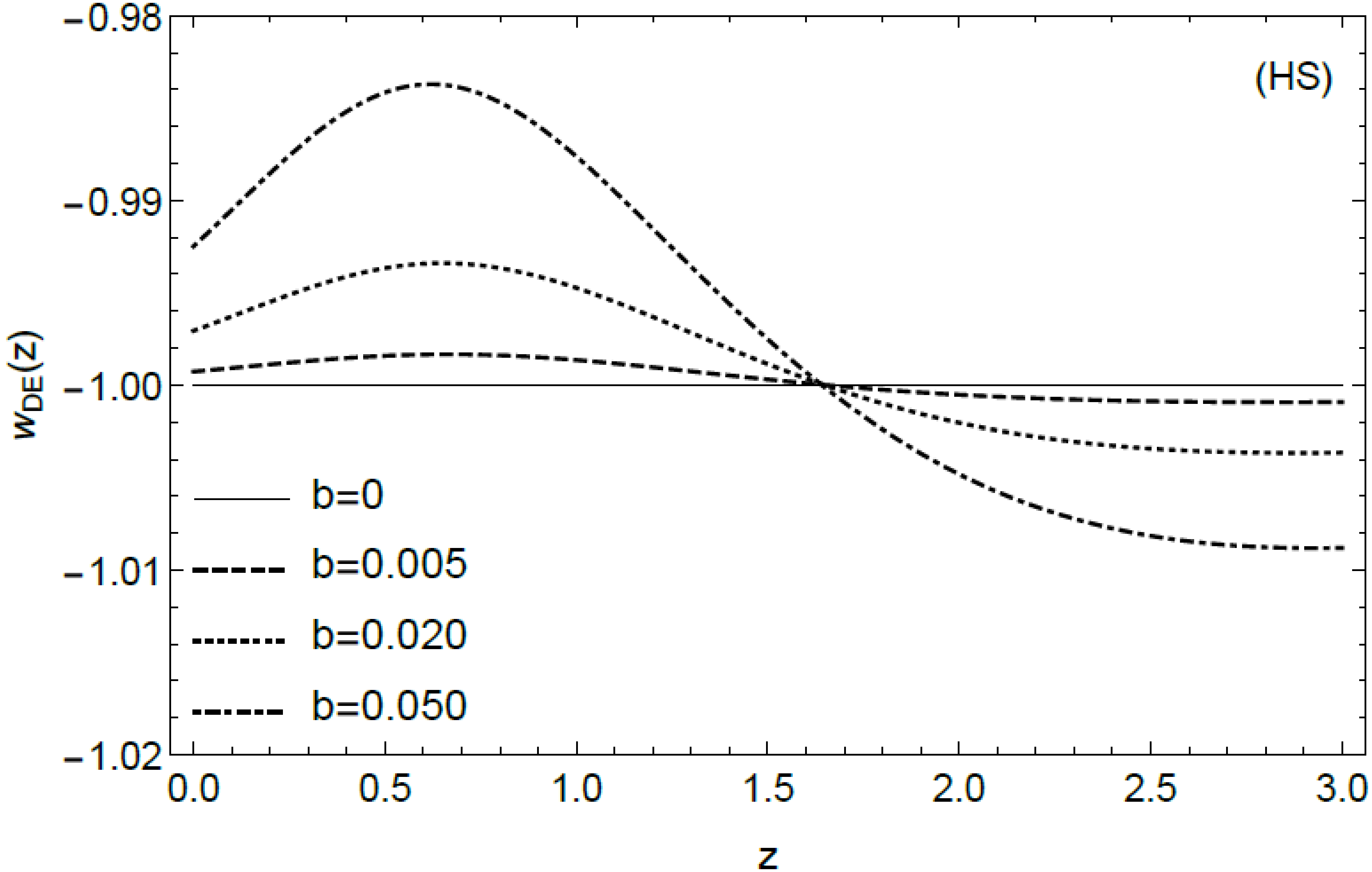
Background. Analytic approximation $b \sim [0.001 - 0.1]$

$$H_{HS}(a)^2 = H_\Lambda(a)^2 + b\,\delta H_1(a)^2 + b^2\,\delta H_2(a)^2 + \cdots$$

arXiv:1302.6501

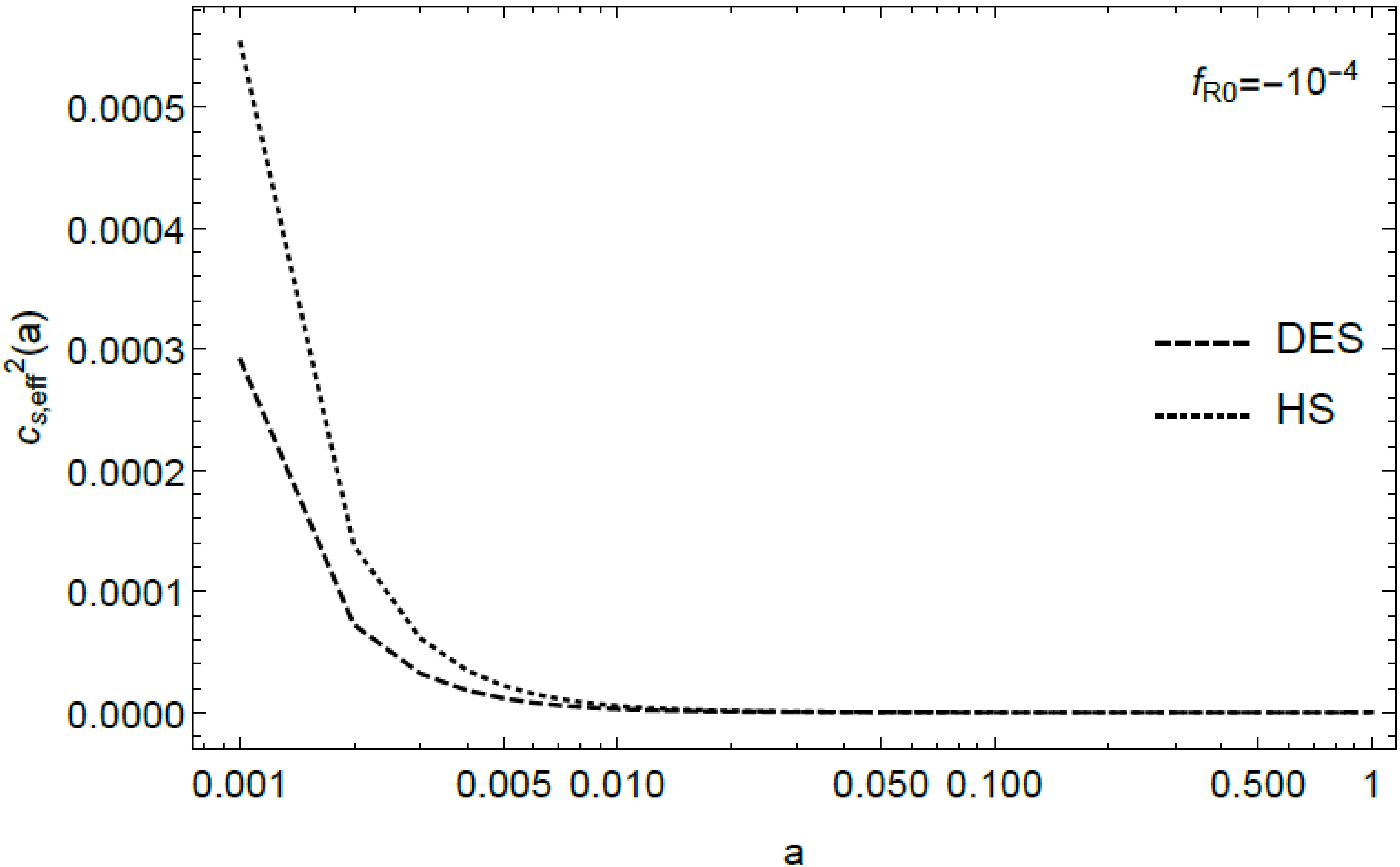
$V \equiv (1 + w)\theta$

DE equation of state



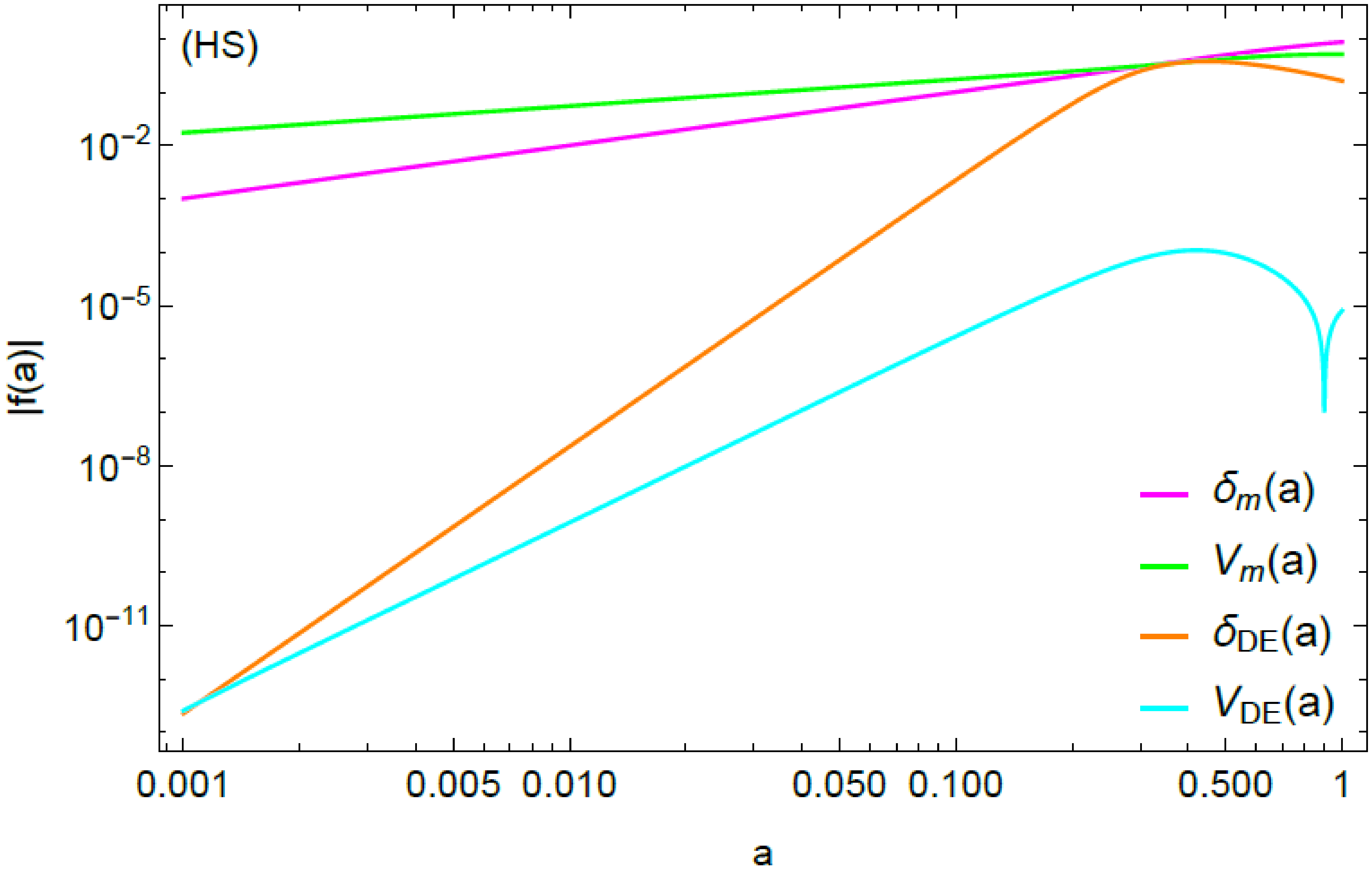
Hu & Sawicki model $\Omega_{m0} = 0.3, \quad b \in [0, 0.05]$

Numerical solution of the evolution equations



$$\Omega_{m0} = 0.3,\ k = 300H_0 \text{ and } f_{R,0} = -10^{-4}$$

Numerical solution of the evolution equations



Horndeski theories

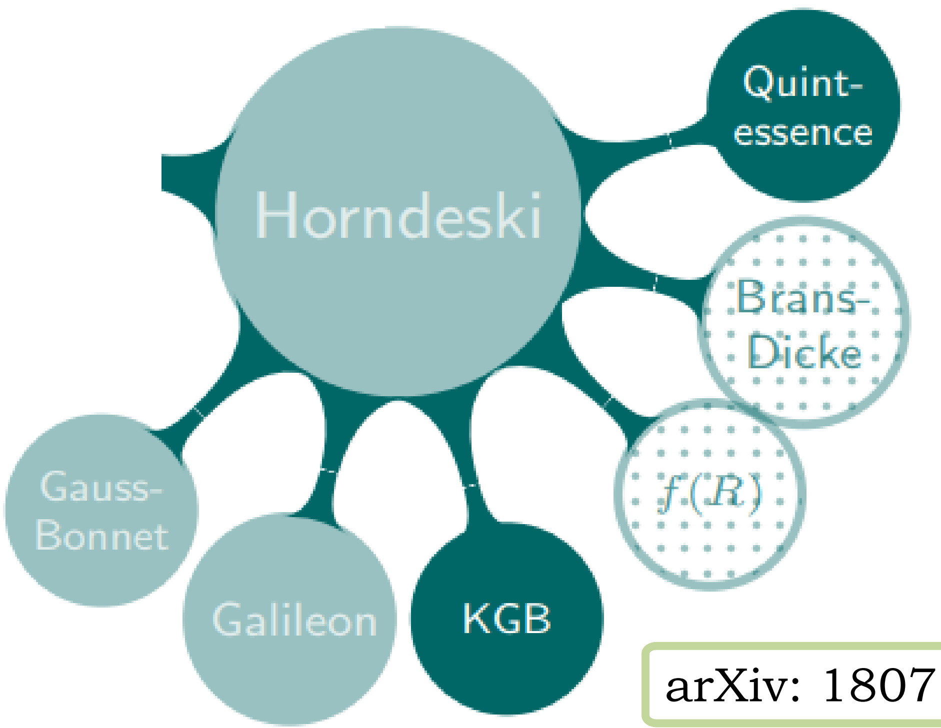
Most general **scalar-tensor theory** whose equations of motion contain derivatives up to **second order**

$$S[g_{\mu\nu},\phi]=\int d^4x\sqrt{-g}\left[\sum_{i=2}^{5}\mathcal{L}_i\left[g_{\mu\nu},\phi\right]+\mathcal{L}_m\left[g_{\mu\nu},\psi_M\right]\right]$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi,X)\\
\mathcal{L}_3 &= -G_3(\phi,X)\,\Box\phi\\
\mathcal{L}_4 &= G_4(\phi,X)\,R+G_{4X}(\phi,X)\left[(\Box\phi)^2-\phi_{;\mu\nu}\phi^{;\mu\nu}\right]\\
\mathcal{L}_5 &= G_5(\phi,X)\,G_{\mu\nu}\phi^{;\mu\nu}-\frac{1}{6}G_{5X}(\phi,X)\left[(\Box\phi)^3+2\phi_{;\mu}^{\nu}\phi_{;\nu}^{\alpha}\phi_{;\alpha}^{\mu}-3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]
\end{aligned}$$

ϕ

Scalar field



$X\equiv-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$

Kinetic term

arXiv: 1807.09241

Horndeski theories

- f(R) theories

$$K = -\frac{Rf_{,R} - f}{2\kappa} \qquad G_4 = \frac{\phi}{2\sqrt{\kappa}} \quad \text{where } \phi \equiv \frac{f_{,R}}{\sqrt{\kappa}}$$

- Kinetic gravity braiding

$$K = K(X) \quad G_3 = G_3(X) \quad G_4 = \frac{1}{2\kappa}$$

- Non-minimal coupling (NMC) model

$$K = \omega(\phi)X - V(\phi) \quad G_4 = \left(\frac{1}{2\kappa} - \frac{\zeta\phi^2}{2}\right) \quad G_3 = 0$$

Higgs inflation $\omega(\phi) = 1,\ V(\phi) = \lambda\left(\phi^2 - \nu^2\right)^2/4.$

Horndeski after GW170817

GRB170817A+GW170817

$$-3\cdot 10^{-15} \le c_g/c - 1 \le 7\cdot 10^{-16}$$

arXiv: 1710.05901

$$c_g = 1 + \alpha_T$$

sound speed tensor perturb.

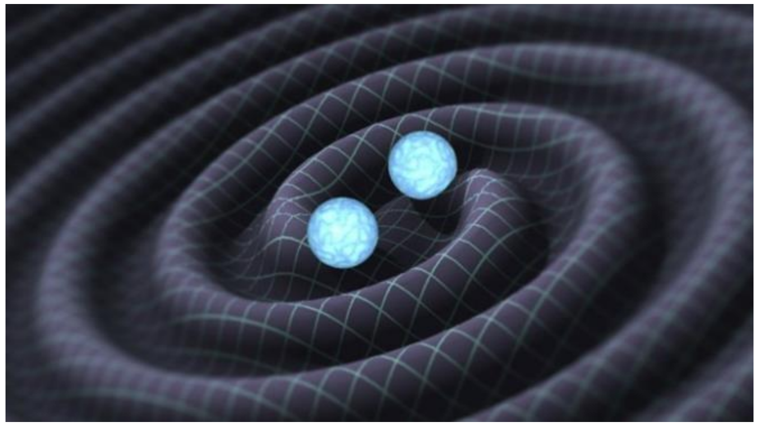
tensor speed excess

$$\ddot{h}_{ij} + (3+\alpha_M)\,H\dot{h}_{ij} + (1+\alpha_T)\,k^2 h_{ij} = 0$$

propagation eq. of GW scalar-tensor gravity

$$G_{4X} \approx 0, \quad G_5 \approx \text{constant}$$

$$ds^2 = -\,(1+2\Psi(\vec{x},t))\,dt^2 + a(t)^2(1+2\Phi(\vec{x},t))d\vec{x}^2$$



$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\,\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)\,R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)\,G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}^{\nu}\phi_{;\nu}^{\alpha}\phi_{;\alpha}^{\mu} - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi\right]$$

More on Horndeski theory

A. Background

$$w_{DE} = \frac{K - \dot{\phi}^2\left(G_{3\phi} + \ddot{\phi}G_{3X}\right) - \left(3H^2 + 2\dot{H}\right)\left(\frac{1}{\kappa} - 2G_4\right) + 2\left(\ddot{\phi} + 2H\dot{\phi}\right)G_{4\phi} + 2\dot{\phi}^2 G_{4\phi\phi}}{\dot{\phi}^2 K_X - K + 3\dot{\phi}^3 H G_{3X} - \dot{\phi}^2 G_{3\phi} + 3H^2\left(\frac{1}{\kappa} - 2G_4\right) - 6H\dot{\phi}G_{4\phi}}$$

B. Perturbations

Gravitational Field Equation

$$\sum_{i=2}^{4} \mathcal{G}^i_{\mu\nu} = \frac{1}{2} T^{(m)}_{\mu\nu}$$

First order Linear Perturbations

$$(0,0) \quad A_1\dot{\Phi} + A_2\dot{\delta\phi} + A_3\frac{k^2}{a^2}\Phi + A_4\Psi + \left(A_6\frac{k^2}{a^2} - \mu\right)\delta\phi - \rho_m\delta_m = 0$$

$$(i,i) \quad B_1\ddot{\Phi} + B_2\ddot{\delta\phi} + B_3\dot{\Phi} + B_4\dot{\delta\phi} + B_5\dot{\Psi} + B_6\frac{k^2}{a^2}\Phi + \left(B_7\frac{k^2}{a^2} + 3\nu\right)\delta\phi + \left(B_8\frac{k^2}{a^2} + B_9\right)\Psi = 0$$

$$(0,i) \quad C_1\dot{\Phi} + C_2\dot{\delta\phi} + C_3\Psi + C_4\delta\phi - \frac{a\rho_m V_m}{k^2} = 0$$

$$(i,j)\ i \neq j \quad G_4\left(\Psi + \Phi\right) + G_{4\phi}\delta\phi = 0$$

Scalar Field Equation

$$\nabla^\mu\left(\sum_{i=2}^{4} J^i_\mu\right) = \sum_{i=2}^{4} P^i_\phi$$

First order Linear Perturbations

$$D_1\ddot{\Phi} + D_2\ddot{\delta\phi} + D_3\dot{\Phi} + D_4\dot{\delta\phi} + D_5\dot{\Psi} + \left(D_7\frac{k^2}{a^2} + D_8\right)\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + \left(D_{10}\frac{k^2}{a^2} + D_{11}\right)\Psi = 0$$

Subhorizon and quasi-static approximation

Gravitational and Scalar Field Equations *First order Linear Perturbations*

$$k^2/a^2 \gg H^2$$

$$A_3\frac{k^2}{a^2}\Phi + A_6\frac{k^2}{a^2}\delta\phi - \kappa\rho_m\delta_m \simeq 0$$
$$B_6\frac{k^2}{a^2}\Phi + B_8\frac{k^2}{a^2}\Psi + B_7\frac{k^2}{a^2}\delta\phi \simeq 0,$$
$$D_7\frac{k^2}{a^2}\Phi + \left(D_9\frac{k^2}{a^2} - M^2\right)\delta\phi + D_{10}\frac{k^2}{a^2}\Psi \simeq 0$$

$$\frac{k^2}{a^2}\Psi = -\frac{\kappa}{2}\frac{G_{\rm eff}}{G_N}\bar{\rho}_m\delta$$

$$\frac{G_{\rm eff}}{G_N} = \frac{2\left[\left(B_6D_9 - B_7^2\right)\frac{k^2}{a^2} - B_6M^2\right]}{\left(A_6^2B_6 + B_6^2D_9 - 2A_6B_7B_6\right)\frac{k^2}{a^2} - B_6^2M^2}$$

$$\frac{k^2}{a^2}\Phi = \frac{\kappa}{2}Q_{\rm eff}\bar{\rho}_m\delta$$

$$\delta\phi = \frac{\left(A_6B_6 - B_6B_7\right)\rho_m\delta_m}{\left(A_6^2B_6 - 2A_6B_6B_7 + B_6^2D_9\right)\frac{k^2}{a^2} - B_6^2M^2}$$

$$Q_{\rm eff} = \frac{2\left[\left(A_6B_7 - B_6D_9\right)\frac{k^2}{a^2} + B_6M^2\right]}{\left(A_6^2B_6 + B_6^2D_9 - 2A_6B_7B_6\right)\frac{k^2}{a^2} - B_6^2M^2}$$

The Effective Fluid Approach

By adding and substracting the Einstein tensor on the LHS of Eq. (1) and moving everything to the RHS we can rewrite the EOM as the usual Einstein equations plus an effective DE fluid along with the usual matter fields.

Gravitational Field Equation Eq.(1)

$$\sum_{i=2}^{4} \mathcal{G}^{i}_{\mu\nu} = \frac{1}{2} T^{(m)}_{\mu\nu}$$

$$G_{\mu\nu} = \kappa \left(T^{(m)}_{\mu\nu} + T^{(DE)}_{\mu\nu} \right) \quad \Rightarrow \quad \kappa T^{(DE)}_{\mu\nu} = G_{\mu\nu} - 2\kappa \sum_{i=2}^{4} \mathcal{G}^{i}_{\mu\nu}$$

$$\begin{aligned}
\frac{\delta P_{DE}}{\bar{\rho}_{DE}} &= (...)\delta\phi + (...)\delta\dot{\phi} + (...)\delta\ddot{\phi} + (...)\Psi + (...)\dot{\Psi} + (...)\Phi + (...)\dot{\Phi} + (...)\ddot{\Phi} \\
\delta_{DE} &= (...)\delta\phi + (...)\delta\dot{\phi} + (...)\Psi + (...)\Phi + (...)\dot{\Phi} \\
V_{DE} &\equiv (1 + w_{DE})\theta_{DE} = (...)\delta\phi + (...)\delta\dot{\phi} + (...)\Psi + (...)\Phi + (...)\dot{\Phi}
\end{aligned}$$

The Effective Fluid Approach

$$\frac{\delta P_{DE}}{\bar\rho_{DE}} = (\ldots)\delta\phi + (\ldots)\dot{\delta\phi} + (\ldots)\ddot{\delta\phi} + (\ldots)\Psi + (\ldots)\dot\Psi + (\ldots)\Phi + (\ldots)\dot\Phi + (\ldots)\ddot\Phi$$

$$\delta_{DE} = (\ldots)\delta\phi + (\ldots)\dot{\delta\phi} + (\ldots)\Psi + (\ldots)\Phi + (\ldots)\dot\Phi$$

$$V_{DE} \equiv (1+w_{DE})\theta_{DE} = (\ldots)\delta\phi + (\ldots)\dot{\delta\phi} + (\ldots)\Psi + (\ldots)\Phi + (\ldots)\dot\Phi$$

Subhorizon and Quasistatic approximation

Horndeski models with DE anisotropic stress

$$\Phi + \Psi = \frac{G_{4\phi}}{G_4}\delta\phi \qquad \pi_{DE} = \frac{\frac{k^2}{a^2}(\Phi+\Psi)}{\kappa\,\bar\rho_{DE}} \simeq \frac{\frac{k^4}{a^4}\mathcal{F}_4^2 B_7 (B_7 - A_6)}{\frac{k^4}{a^4}\mathcal{F}_7 + \frac{k^2}{a^2}\mathcal{F}_8 + \mathcal{F}_9}\delta_{DE}$$

$$\frac{\delta P_{DE}}{\bar\rho_{DE}} \simeq \frac{1}{3\mathcal{F}_4}\frac{\frac{k^4}{a^4}\mathcal{F}_1 + \frac{k^2}{a^2}\mathcal{F}_2 + \mathcal{F}_3}{\frac{k^4}{a^4}\mathcal{F}_5 + \frac{k^2}{a^2}\mathcal{F}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{\frac{k^4}{a^4}\mathcal{F}_7 + \frac{k^2}{a^2}\mathcal{F}_8 + \mathcal{F}_9}{\frac{k^4}{a^4}\mathcal{F}_5 + \frac{k^2}{a^2}\mathcal{F}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$V_{DE} \simeq a\frac{\frac{k^2}{a^2}\mathcal{F}_{10} + \mathcal{F}_{11}}{\frac{k^2}{a^2}\mathcal{F}_5 + \mathcal{F}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$c^2_{s,DE} \equiv \frac{\delta P_{DE}}{\delta\rho_{DE}} = \frac{1}{3}\frac{\frac{k^4}{a^4}\mathcal{F}_1 + \frac{k^2}{a^2}\mathcal{F}_2 + \mathcal{F}_3}{\frac{k^4}{a^4}\mathcal{F}_7 + \frac{k^2}{a^2}\mathcal{F}_8 + \mathcal{F}_9}$$

f(R)

Horndeski models with NON DE anisotropic stress

$$\Phi = -\Psi \qquad \pi_{DE} = 0$$

$$\frac{\delta P_{DE}}{\bar\rho_{DE}} \simeq \frac{1}{3}\frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_2 + \hat{\mathcal{F}}_3}{\frac{k^4}{a^4}\hat{\mathcal{F}}_5 + \frac{k^2}{a^2}\hat{\mathcal{F}}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$\delta_{DE} \simeq \frac{\frac{k^4}{a^4}\hat{\mathcal{F}}_7 + \frac{k^2}{a^2}\hat{\mathcal{F}}_8 + \hat{\mathcal{F}}_9}{\frac{k^4}{a^4}\hat{\mathcal{F}}_5 + \frac{k^2}{a^2}\hat{\mathcal{F}}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$V_{DE} \simeq a\frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_{10} + \hat{\mathcal{F}}_{11}}{\frac{k^2}{a^2}\hat{\mathcal{F}}_5 + \hat{\mathcal{F}}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$c^2_{s,DE} = \frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_2 + \hat{\mathcal{F}}_3}{\frac{k^4}{a^4}\hat{\mathcal{F}}_7 + \frac{k^2}{a^2}\hat{\mathcal{F}}_8 + \hat{\mathcal{F}}_9}$$

Quintessence, K-essence
Kinetic Gravity Braiding
Designer Model (HDES)

Designer model (HDES)

Background exactly equal to that of ΛCDM model but perturbations given by the Horndeski theory

Modified Friedmann Equation

$$-H(a)^2 - \frac{K(X)}{3} + H_0^2\Omega_m(a) + 2\sqrt{2}X^{3/2}H(a)G_{3X} + \frac{2}{3}XK_X = 0$$

Scalar Field Conservation Equation

$$\frac{J_c}{a^3} - 6XH(a)G_{3X} - \sqrt{2}\sqrt{X}K_X = 0$$

$\phi \rightarrow \phi + c$

$(G_{3X}(X), K(X), H(a))$ ➡ H=H(X) and solve the system

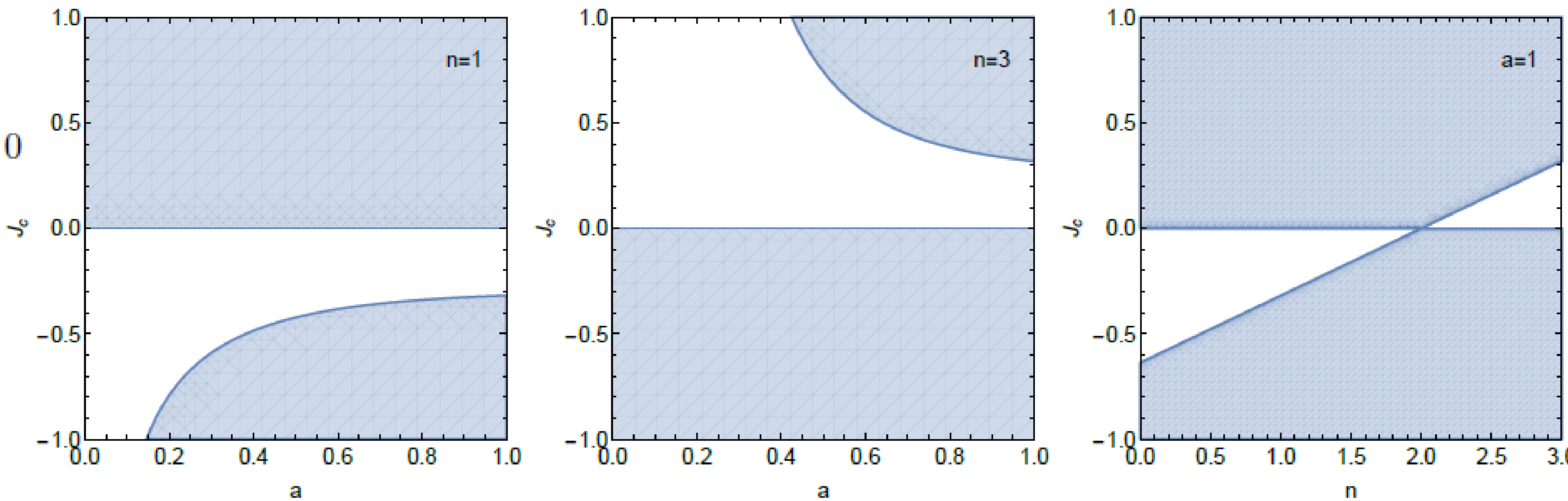
Family of Designer Models

$$K(X) = -3H_0^2\Omega_{\Lambda,0} + \frac{J_c\sqrt{2X}H(X)^2}{H_0^2\Omega_{m,0}} - \frac{J_c\sqrt{2X}\Omega_{\Lambda,0}}{\Omega_{m,0}} \qquad G_{3X}(X) = -\frac{2J_cH'(X)}{3H_0^2\Omega_{m,0}}$$

HDES

$$X = \frac{c_0}{H(a)^n} \qquad K(X) = \frac{\sqrt{2}J_c c_0^{2/n}X^{\frac{1}{2}-\frac{2}{n}}}{H_0^2\Omega_{m,0}} - 3H_0^2\Omega_{\Lambda,0} - \frac{\sqrt{2}J_c\sqrt{X}\Omega_{\Lambda,0}}{\Omega_{m,0}} \qquad G_3(X) = -\frac{2J_c c_0^{1/n}X^{-1/n}}{3H_0^2\Omega_{m,0}}$$

$\alpha_K + \frac{3}{2}\alpha_B^2 > 0$



Numerical solution

A) Full-DES. Numerical solution of the full system of equations.

B) Eff. Fluid. Numerical solution of the effective fluid approach.

C) ODE_Geff. Numerical solution of the growth factor equation.

D) The ΛCDM model.

A)

$$(0,0) \quad A_1\dot{\Phi} + A_2\dot{\delta\phi} + A_3\frac{k^2}{a^2}\Phi + A_4\Psi + \left(A_6\frac{k^2}{a^2} - \mu\right)\delta\phi - \rho_m\delta_m = 0$$

$$(i,i) \quad B_1\ddot{\Phi} + B_2\ddot{\delta\phi} + B_3\dot{\Phi} + B_4\dot{\delta\phi} + B_5\dot{\Psi} + B_6\frac{k^2}{a^2}\Phi + \left(B_7\frac{k^2}{a^2} + 3\nu\right)\delta\phi + \left(B_8\frac{k^2}{a^2} + B_9\right)\Psi = 0$$

$$(0,i) \quad C_1\dot{\Phi} + C_2\dot{\delta\phi} + C_3\Psi + C_4\delta\phi - \frac{a\rho_m V_m}{k^2} = 0$$

$$(i,j)\ i \neq j \quad G_4(\Psi + \Phi) + G_{4\phi}\delta\phi = 0$$

B) $\nabla_\nu T^{\mu\nu} = 0 \Longrightarrow$

$$\delta' = -3(1+w)\Phi' - \frac{V}{a^2H} - \frac{3}{a}\left(\frac{\delta P}{\bar{\rho}} - w\delta\right)$$

$$V' = -(1-3w)\frac{V}{a} + \frac{k^2}{a^2H}\frac{\delta P}{\bar{\rho}} + (1+w)\frac{k^2}{a^2H}\Psi - \frac{2}{3}\frac{k^2}{a^2H}\pi$$

C)

$$\delta_m''(a) + \left(\frac{3}{a} + \frac{H'(a)}{H(a)}\right)\delta_m'(a) - \frac{3}{2}\frac{\Omega_{m0}G_{\text{eff}}/G_N}{a^5H(a)^2/H_0^2}\delta_m(a) = 0$$

D)

$$\delta(a) = a\ {}_2F_1\left[-\frac{1}{3w}, \frac{1}{2} - \frac{1}{2w}; 1 - \frac{5}{6w}; a^{-3w}(1-\Omega_{\rm m}^{-1})\right]$$

Growth of matter perturbations

Define growth rate f(a): $$f(a) = \frac{d\log\delta_m}{d\log a}$$

However, the measurable quantity is fσ8=f(a)* σ 8(a)

$$f\sigma_8(a) = a\frac{\delta'_m(a)}{\delta_m(1)}\sigma_{8,0}$$

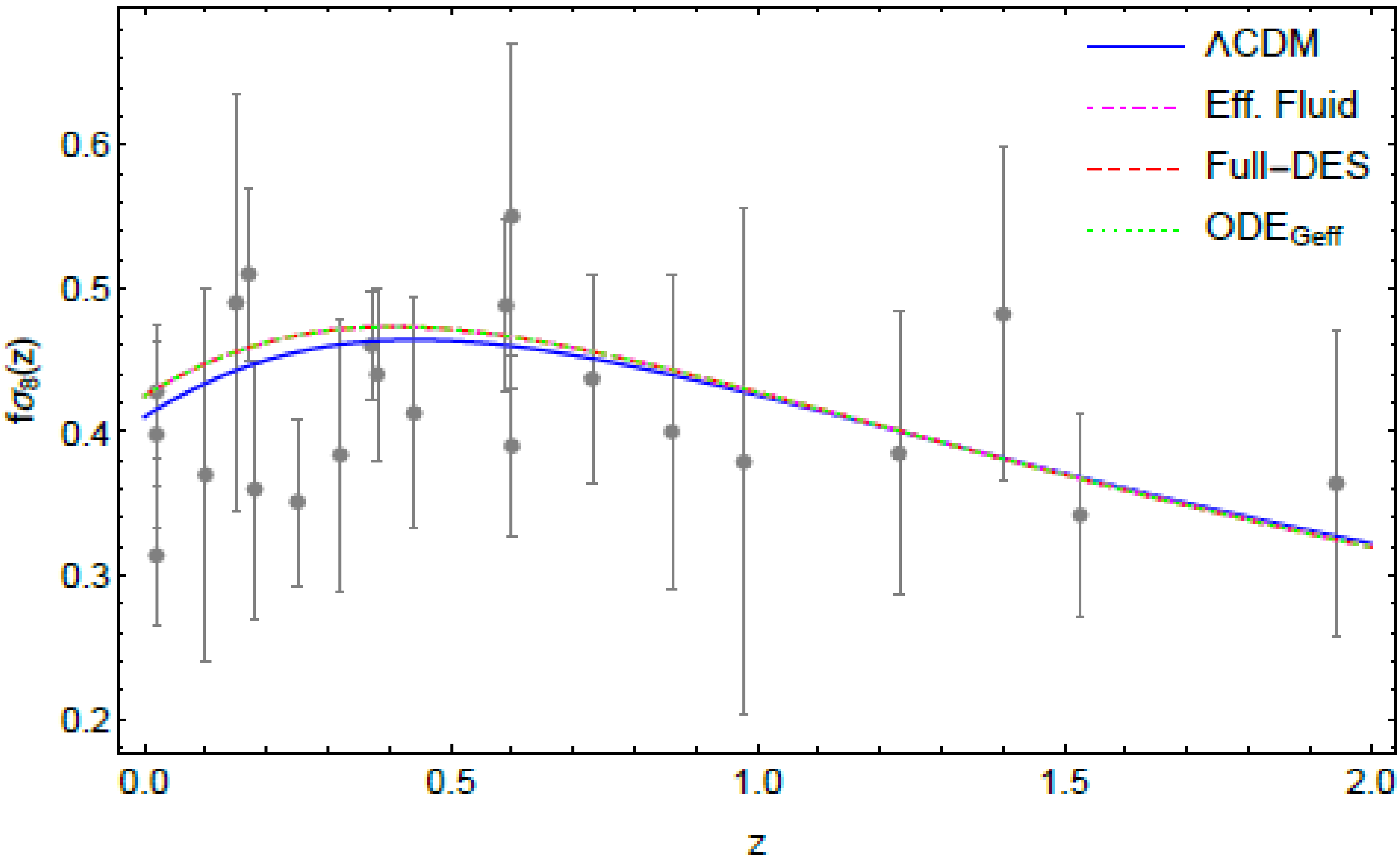
where $$\sigma_8(a) = \frac{\delta_m(a)}{\delta_m(1)}\sigma_{8,0}$$ $R = 8h^{-1}\mathrm{Mpc}$

Redshift dependent rms fluctuation of the linear density field with spheres of radius R.

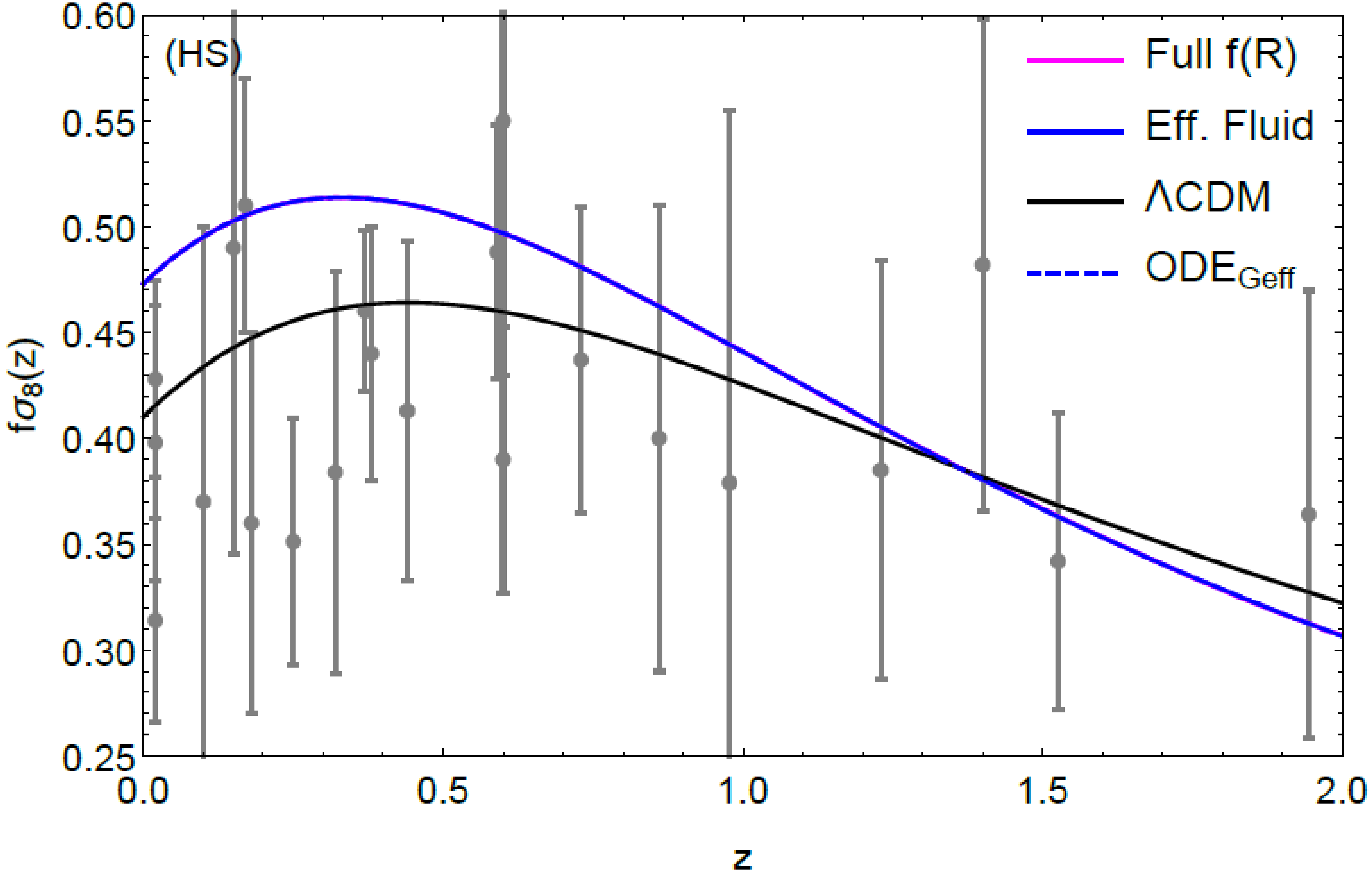
$\tilde{J}_c = J_c/H_0$ and $\tilde{c}_0 = c_0/H_0^{n+2} = 1$

$\Omega_{m,0} = 0.3,\ k = 300H_0$ and $\sigma_{8,0} = 0.8$ $\quad n = 2\ \ \tilde{J}_c = 5\cdot 10^{-2}$

Growth of matter perturbations



Numerical solution of the evolution equations



Future surveys: Euclid and LSST → **Constrain with higher accuracy**

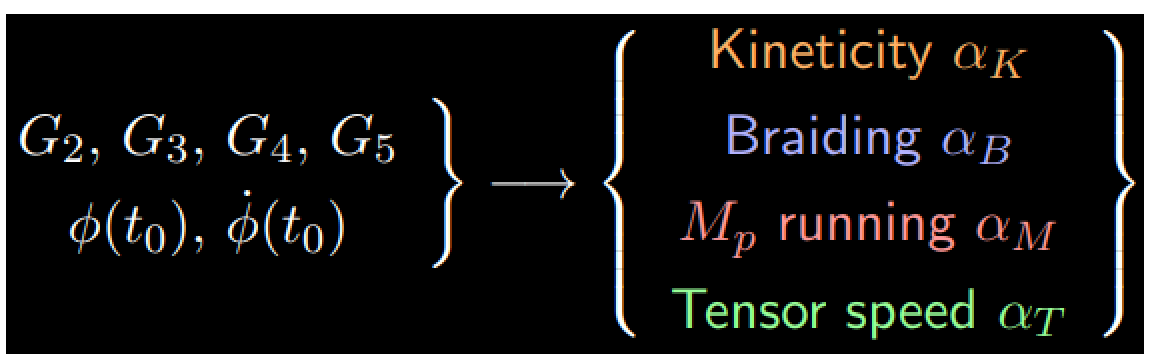
HDES: Modifications to CLASS

EFCLASS
$$\begin{cases} V' = -(1-3w)\dfrac{V}{a} + \dfrac{k^2}{a^2H}\dfrac{\delta P}{\bar\rho} + (1+w)\dfrac{k^2}{a^2H}\Psi - \dfrac{2}{3}\dfrac{k^2}{a^2H}\pi \\ \pi_{DE} = 0 \end{cases}$$

Using
$$\frac{\delta P_{DE}}{\bar\rho_{DE}} \simeq \frac{1}{3}\frac{\frac{k^2}{a^2}\hat{\mathcal{F}}_2 + \hat{\mathcal{F}}_3}{\frac{k^4}{a^4}\hat{\mathcal{F}}_5 + \frac{k^2}{a^2}\hat{\mathcal{F}}_6}\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

$$V_{DE} \simeq \left(-\frac{14\sqrt{2}}{3}\Omega_{m,0}^{-3/4}\tilde{J}_c\, H_0\, a^{1/4}\right)\frac{\bar\rho_m}{\bar\rho_{DE}}\delta_m$$

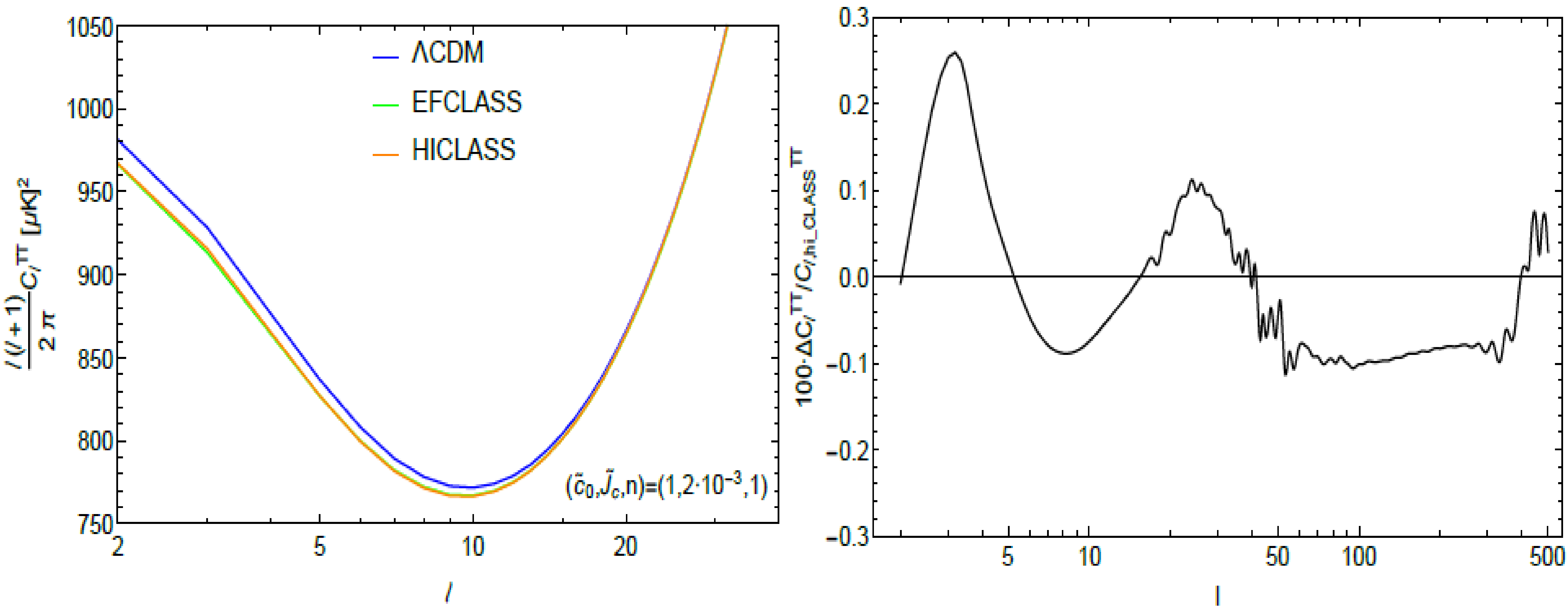
$$\tilde{J}_c = J_c/H_0 \text{ and } \tilde{c}_0 = c_0/H_0^{n+2} = 1$$



$$\begin{aligned} M_*^2 &\equiv 1 \\ \alpha_M &\equiv \frac{d\ln M_*^2}{d\ln a} = 0 \\ \alpha_K &\equiv -\frac{4\sqrt{2}\sqrt{c_0}J_c(n-2)H(a)^{-\frac{n}{2}}}{H_0^2 n^2\Omega_{m,0}} \\ \alpha_B &\equiv \frac{4\sqrt{2}\sqrt{c_0}J_cH(a)^{-\frac{n}{2}}}{3H_0^2 n\Omega_{m,0}} \\ \alpha_T &\equiv 0 \end{aligned}$$

hi_class implements Horndeski's theory in the modern **Cosmic Linear Anisotropy Solving System**. It can be used to compute any linear observable in seconds, including cosmological distances, CMB, matter power and number counts spectra.

low-l multipoles TT CMB spectrum



$$\Omega_{m,0} = 0.3,\ n_s = 1,\ A_s = 2.3 \cdot 10^{-9}\ ,\ h = 0.7 \text{ and } (\tilde{c}_0, \tilde{J}_c, n) = (1, 2 \cdot 10^{-3}, 1)$$

Data for the MCMC

$$L_{\text{tot}} = L_{\text{SnIa}} \times L_{\text{BAO}} \times L_{\text{H(z)}} \times L_{\text{CMB}} \times L_{\text{growth}}$$

$$\chi^2_{\text{tot}} = \chi^2_{\text{SnIa}} + \chi^2_{\text{BAO}} + \chi^2_{\text{H(z)}} + \chi^2_{\text{cmb}} + \chi^2_{\text{growth}}$$

1048 data points from Pantheon, 3 from the CMB shift parameters, 10 from the BAO measurements, 22 from the growth and 36 H(z) points
Total: N=1118.

z	$H(z)$	σ_H	Ref.	z	$H(z)$	σ_H	Ref.
0.07	69.0	19.6	[110]	0.48	97.0	62.0	[111]
0.09	69.0	12.0	[111]	0.57	96.8	3.4	[91]
0.12	68.6	26.2	[110]	0.593	104.0	13.0	[112]
0.17	83.0	8.0	[111]	0.60	87.9	6.1	[93]
0.179	75.0	4.0	[112]	0.68	92.0	8.0	[112]
0.199	75.0	5.0	[112]	0.73	97.3	7.0	[93]
0.2	72.9	29.6	[110]	0.781	105.0	12.0	[112]
0.27	77.0	14.0	[111]	0.875	125.0	17.0	[112]
0.28	88.8	36.6	[110]	0.88	90.0	40.0	[111]
0.35	82.7	8.4	[113]	0.9	117.0	23.0	[111]
0.352	83.0	14.0	[112]	1.037	154.0	20.0	[112]
0.3802	83.0	13.5	[108]	1.3	168.0	17.0	[111]
0.4	95.0	17.0	[111]	1.363	160.0	33.6	[114]
0.4004	77.0	10.2	[108]	1.43	177.0	18.0	[111]
0.4247	87.1	11.2	[108]	1.53	140.0	14.0	[111]
0.44	82.6	7.8	[93]	1.75	202.0	40.0	[111]
0.44497	92.8	12.9	[108]	1.965	186.5	50.4	[114]
0.4783	80.9	9.0	[108]	2.34	222.0	7.0	[115]

z	$f\sigma_8(z)$	$\sigma_{f\sigma_8}(z)$	$\Omega^{\text{ref}}_{m,0}$	Ref.
0.02	0.428	0.0465	0.3	[116]
0.02	0.398	0.065	0.3	[117],[118]
0.02	0.314	0.048	0.266	[119],[118]
0.10	0.370	0.130	0.3	[120]
0.15	0.490	0.145	0.31	[121]
0.17	0.510	0.060	0.3	[101]
0.18	0.360	0.090	0.27	[122]
0.38	0.440	0.060	0.27	[122]
0.25	0.3512	0.0583	0.25	[123]
0.37	0.4602	0.0378	0.25	[123]
0.32	0.384	0.095	0.274	[124]
0.59	0.488	0.060	0.307115	[125]
0.44	0.413	0.080	0.27	[93]
0.60	0.390	0.063	0.27	[93]
0.73	0.437	0.072	0.27	[93]
0.60	0.550	0.120	0.3	[126]
0.86	0.400	0.110	0.3	[126]
1.40	0.482	0.116	0.27	[127]
0.978	0.379	0.176	0.31	[128]
1.23	0.385	0.099	0.31	[128]
1.526	0.342	0.070	0.31	[128]
1.944	0.364	0.106	0.31	[128]

HDES MCMC

Model	Ω_{m0}	$100\Omega_b h^2$	$\bar{J}_c$	h	σ_8
Best-fit values					
ΛCDM	0.311 ± 0.006	2.243 ± 0.014	0	0.680 ± 0.004	0.758 ± 0.025
HDES	0.313 ± 0.006	2.240 ± 0.014	-0.309 ± 0.244	0.678 ± 0.004	0.911 ± 0.068

Model	χ^2	AIC	ΔAIC
ΛCDM	1087.64	1095.68	0
HDES	1086.30	1096.35	0.678

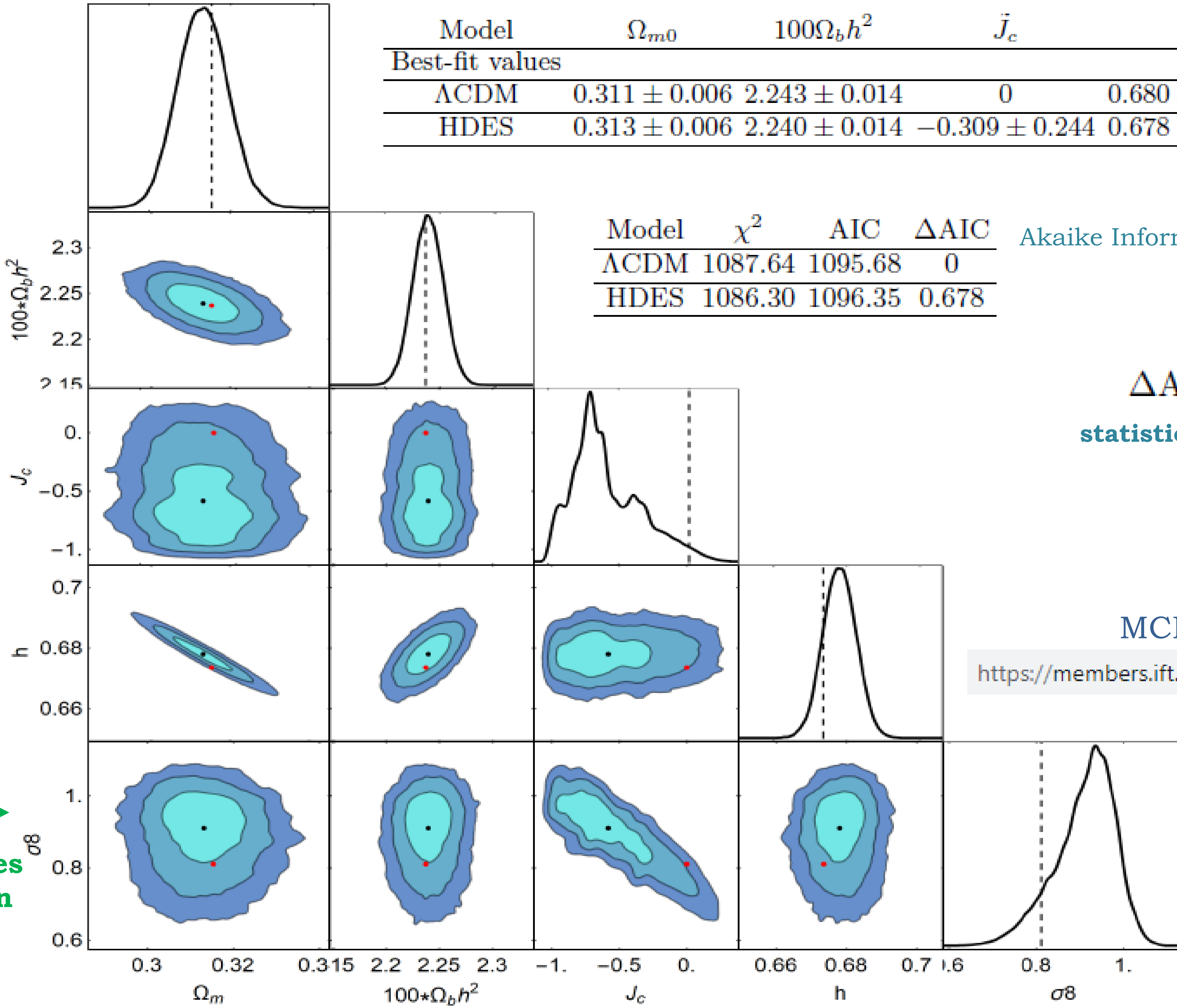
Akaike Information Criterion (AIC)

$\Delta\text{AIC} \lesssim 2$

statistically equivalent

MCMC codes

https://members.ift.uam-csic.es/savvas.nesseris/



alleviates tension

Conclusions

- **Theoretical expressions** for the effective dark energy pressure, velocity and sound speed (Effective Fluid Approach).
- Presented **Designer Horndeski** models (HDES).
- **Numerical solutions** for HDES in Good agreement with $f\sigma_8$ data.
- Our **EFCLASS** modification is accurate to the level of ~0.1%.
- **MCMC** on our HDES model. Both models are statistically consistent.

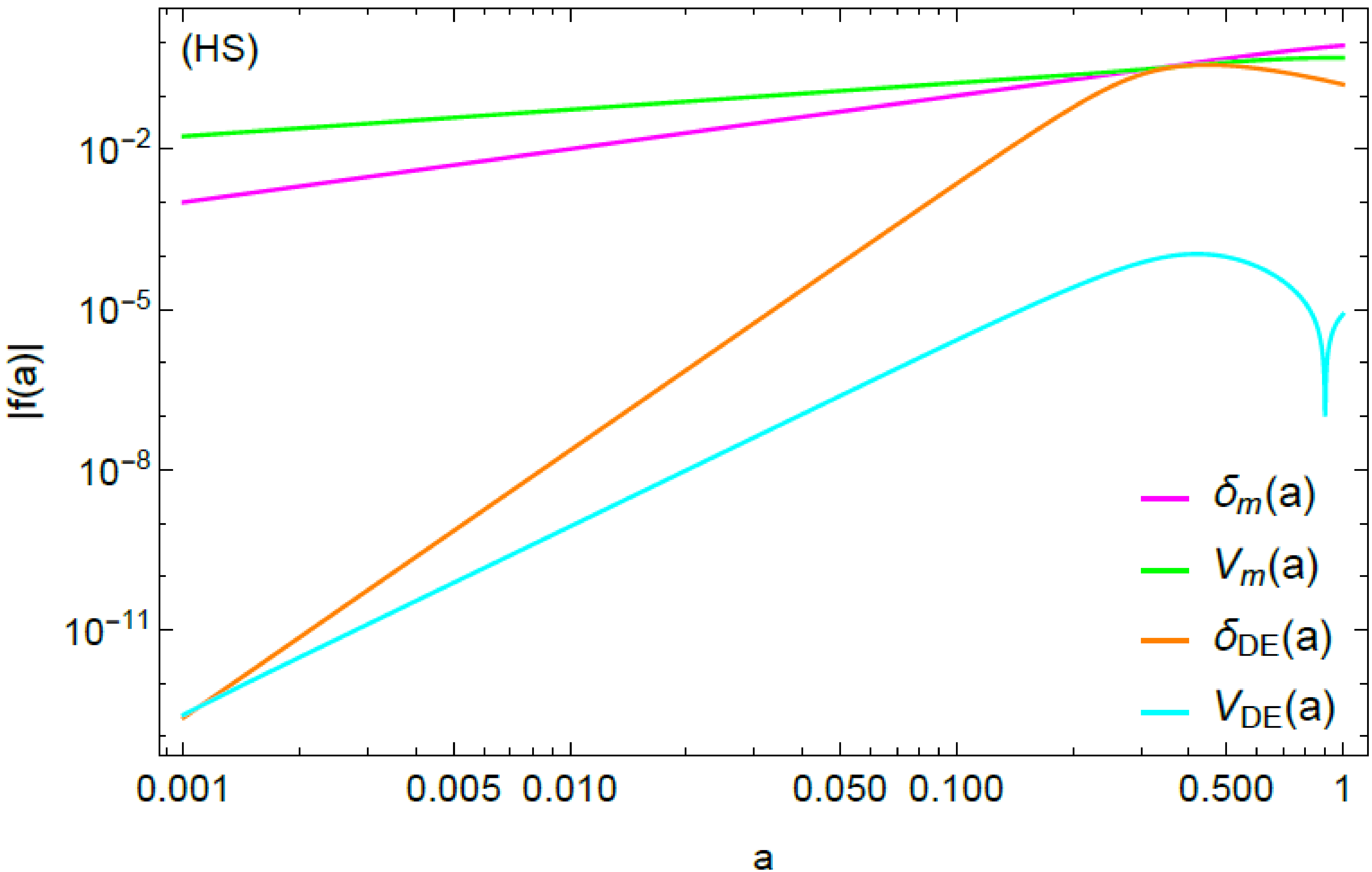
Thank you for your attention!

arXiv:1811.02469 R.Arjona, W.Cardona, S.Nesseris

arXiv:1904.06294 R.Arjona, W.Cardona, S.Nesseris

Back-up slides

Numerical solution of the evolution equations



Why GR is not renomalizable (only at first loop)

$$D = d - N_F\left(\frac{d-1}{2}\right) - N_B\left(\frac{d-2}{2}\right) - \sum_v [\lambda_v]$$

Primitive degree of divergence

Coupling dimension

$[\lambda_v] > 0$	Super-renormalizable
$[\lambda_v] = 0$	Renormalizable
$[\lambda_v] < 0$	Non-renormalizable

GR ⟹ $V = G_N \frac{m_1 m_2}{r}, \quad [G_N] = -2$

Fermi Th. ⟹ $G_F \sim \frac{1}{M_W^2}, \quad [G_F] = -2$

Renormalizing GR to first loop order for Ricci scalar

$$R \longrightarrow a_0(g_{\mu\nu})R + a_1(g_{\mu\nu}) + a_2(g_{\mu\nu})$$

$$a_0(g_{\mu\nu}) = 1$$

$$a_1(g_{\mu\nu}) = \left(\frac{1}{6} - \xi\right) R$$ → Conformal coupling and Gauss Bonnet term

$$a_2(g_{\mu\nu}) = \frac{1}{180} R_{\alpha\beta;\delta} R^{\alpha\beta;\delta} - \frac{1}{180} R^{\alpha\beta} R_{\alpha\beta} - \frac{1}{6}\left(\frac{1}{5} - \xi\right) \Box R + \frac{1}{2}\left(\frac{1}{6} - \xi\right)^2 R^2$$

Higher order corrections to GR

Quantum fields in curved space. Birrel and Davies.

RMS

$$\sigma_8(a) = \frac{\delta_m(a)}{\delta_m(1)}\sigma_{8,0}$$

$$\sigma^2(R,z) = \int_0^\infty W^2(kR)\Delta^2(k,z)\frac{dk}{k}$$

$$W(kR) = 3\left(\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2}\right)$$

$$\Delta^2(kz) = 4\pi k^3 P_\delta(k,z)$$

C L A S S
the Cosmic Linear Anisotropy Solving System

The purpose of CLASS is to simulate the evolution of linear perturbations in the universe and to compute CMB and large scale structure observables.

Akaike Information Criterion (AIC)

Assuming Gaussian errors the AIC estimator is given by

$$\mathrm{AIC} = -2 \ln \mathcal{L}_{\max} + 2k_p + \frac{2k_p(k_p+1)}{N_{\mathrm{dat}} - k_p - 1}$$

k_p number of free parameters

smaller value implies a better fit to the data

$$\Delta\mathrm{AIC} = \mathrm{AIC}_{\mathrm{model}} - \mathrm{AIC}_{\mathrm{min}}$$

To compare different models

$4 < \Delta\mathrm{AIC} < 7$ Positive evidence against the model with higher value

$\Delta\mathrm{AIC} \geq 10$ Strong evidence

$\Delta\mathrm{AIC} \leq 2$ Consistency of the two models

$$c_g^2 = \frac{G_4 - XG_{5\phi} - XG_{5X}\ddot{\phi}}{G_4 - 2XG_{4X} - X\left(G_{5X}\dot{\phi}H - G_{5\phi}\right)}$$

$$-3 \cdot 10^{-15} \le c_g/c - 1 \le 7 \cdot 10^{-16}$$

$$c_g = 1 + \alpha_T$$

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [47] Brans-Dicke/$f(R)$ [48, 49] Kinetic Gravity Braiding [51]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [50] $G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [5], $f(\phi)$·Gauss-Bonnet [53]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

Ezquiaga et al. 1710.05901

Growth rate data

Surveys can provide measurements of the perturbations in terms of the galaxy density δg:

matter perturbations

$$\delta_g = b\delta_m$$

bias parameter

Early measurements: $\beta = \frac{f}{b}$ $b \in [1,3]$ Unreliable datasets of β(z)

$$f\sigma_8(a) = a\frac{\delta'_m(a)}{\delta_m(1)}\sigma_{8,0}$$

→ **Independent of the bias**

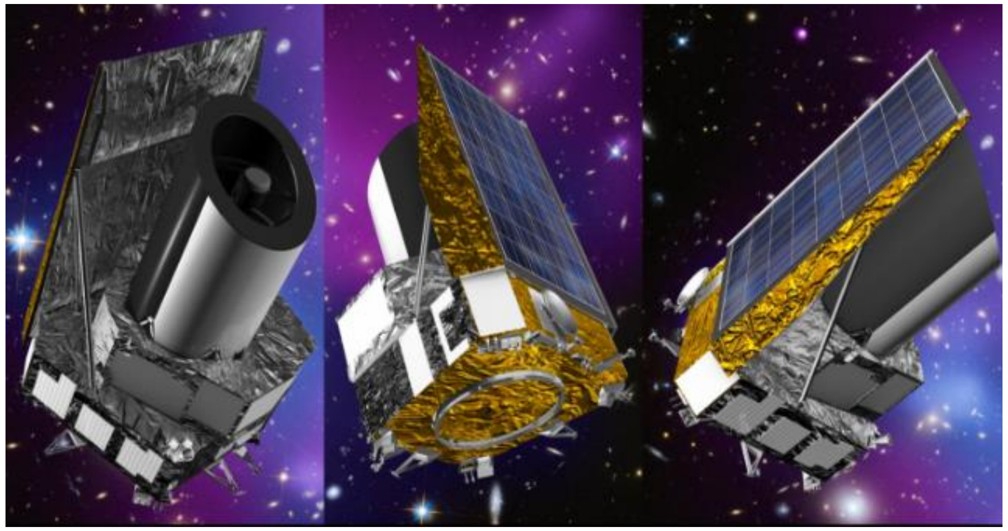
Nesseris et al. 1703.10538

Future surveys

Euclid Consortium

A space mission to map the Dark Universe

Launch is planned for 2021



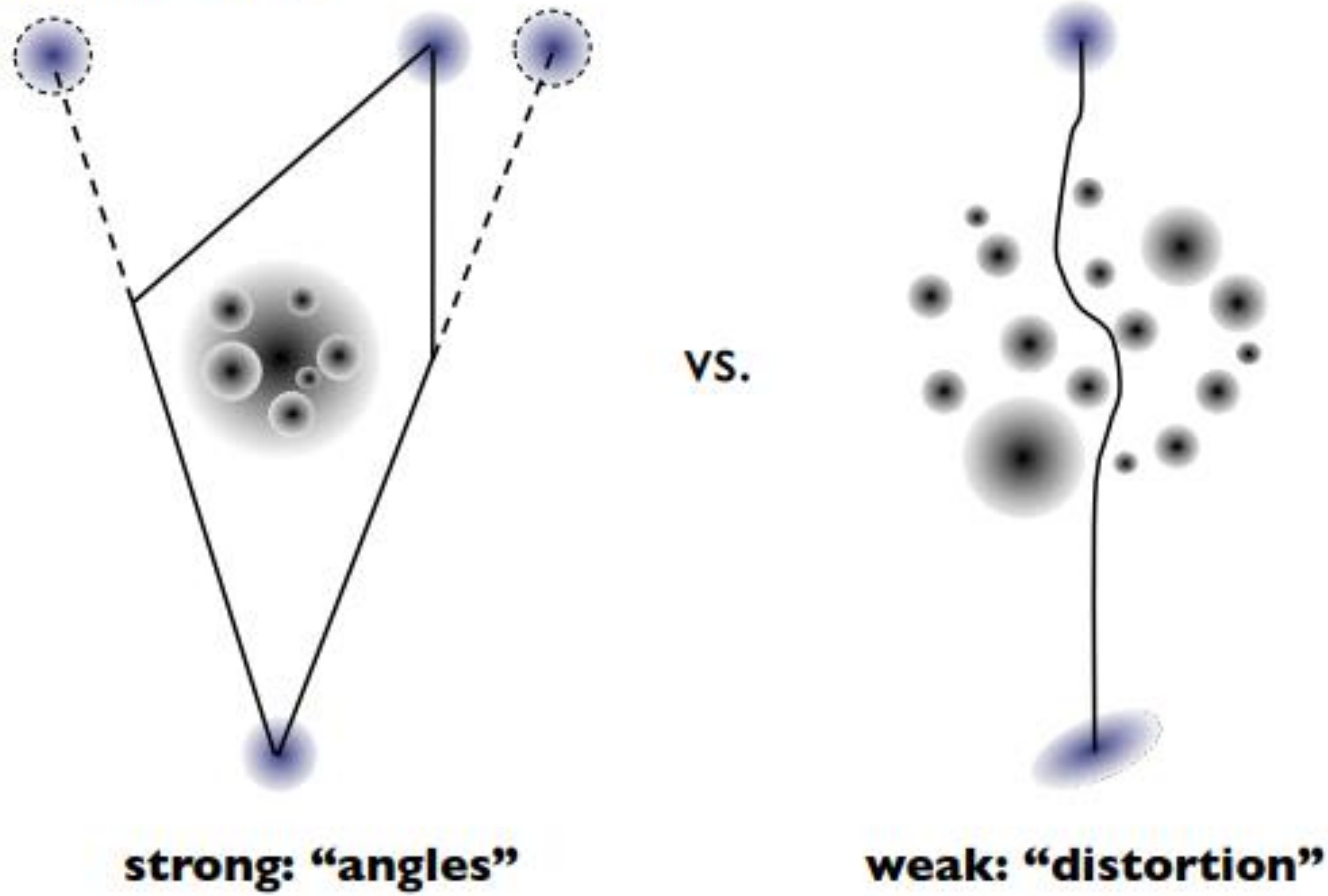
Science operations starts in 2023

LSST

Large Synoptic Survey Telescope

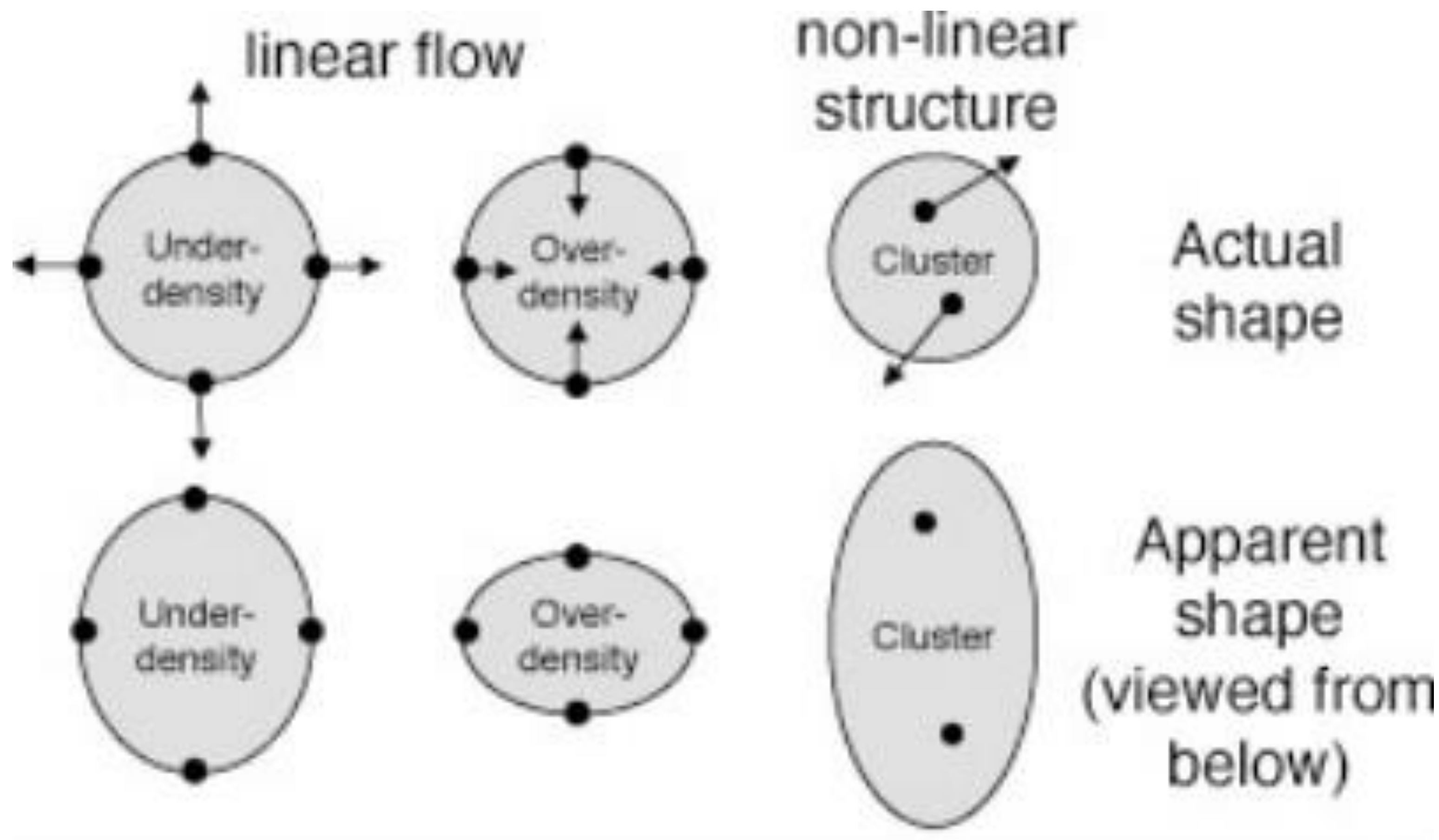
Opening a Window of Discovery on the Dynamic Universe

Weak Gravitational Lensing



- weak lensing
  - lensing via large-scale structure
    (→ weak distortion and magnification)

Redshift space distortions



Redshift-space distortions are an effect in observational cosmology where the spatial distribution of galaxies appears squashed and distorted when their positions are plotted in redshift-space (i.e. as a function of their redshift) rather than in real-space (as a function of their actual distance).
The effect is due to the peculiar velocities of the galaxies causing a Doppler shift in addition to the redshift caused by the cosmological expansion.