Analogue Gravity and the de Broglie wave: a Missed Opportunity

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Abstract

In scientific explanation, the objective of analogical reasoning is to assess the plausibility of some theoretically predicted, but empirically inaccessible, phenomenon by investigating an analogue in which the corresponding feature *is* empirically accessible. The well-known example from analogue gravity is Hawking radiation, which is predicted theoretically and would have significant consequences for the thermodynamics of black holes, but has been thought to be too weak for detection by methods currently available.

A related concern is the possibility of a theory of quantum gravity that would unite the currently disparate theories of gravity and quantum mechanics. It is therefore a significant deficiency in analogues of gravity described to date that they fail to replicate the de Broglie wave. It was de Broglie's prediction of this "matter wave" in 1923 that allowed all particles, whether massive or massive, to be treated in terms of evolving wave characteristics. Were it not for the wave characteristics described by the de Broglie wave there would be no quantum mechanics, and nor therefore, could there be a theory of quantum gravity.

While the ontology of this mysterious wave has been a matter for debate, it is evidently a relativistic effect that arises when a massive particle is considered from an inertial frame in which the particle is moving. It will thus be instructive to consider why this wave has gone missing from a subset of these analogues [1-2] that seek to simulate the Minkowski metric.

In these particular analogues, massive particles are modelled as superpositions of counter-propagating sound waves. The authors of these models show that if an object formed from these sonic particles is to move through the subsisting medium, it must transform between inertial frames in accordance with a Lorentz transformation based on a Lorentz factor,

$$\gamma = (1 - v^2/c_s^2)^{-\frac{1}{2}}, \tag{1}$$

where the velocity c_s is that of sound rather than that of light.

Yet in the rest frame of the subsisting medium, which in the case of sound is typically air, those counter-propagating sound waves constitute a standing wave,

and it is easily shown, and is indeed well-known, that the Lorentz transformation of a standing wave does induce a waveform with the wave characteristics of the de Broglie wave. The standing wave becomes a travelling wave which is contracted in length in the direction of travel in the manner of the Fitzgerald-Lorentz contraction, and is subject to a modulation (a dephasing or beating) that has the form of the de Broglie wave and which implies a failure of simultaneity of exactly the magnitude described by the de Broglie wave.

On investigating why the de Broglie wave fails to emerge in these sonic analogues, it becomes apparent that in simulating the Lorentz transformation, the authors of these models have effectively changed horses in midstream. They commence with standing waves, but purport to establish as an intermediate step that an array of such standing waves acts like a rigid body, which allows them to proceed, very much as Einstein did in 1905, to deduce the Lorentz transformation from an analysis of ideal measuring rods and clocks.

There authors of these analogues were justified in assuming that solid matter might be appropriately modelled by underlying wave-like influences of some fundamental velocity. Indeed, in his famous paper of 1905, Einstein did himself assimilate the behaviour of rods and clocks to that of counter-propagating light waves of velocity c.

But Einstein was unaware in 1905 of the de Broglie wave and of the underlying wave-like nature of matter. He could not take the further step, suggested by these sonic analogues, of concluding that such rods and clocks must necessarily transform in the manner of counter-propagating waves of velocity c because they are themselves constituted from underlying wave-like influences of that velocity.

My contention will be, as I have argued elsewhere [3] and [4], that in these sonic simulations of the Minkowski metric, an opportunity has been missed, not only to explain the de Broglie wave from elementary wave theory, but to demonstrate a common origin for the Minkowski metric and quantum mechanics in the wave structure of matter, which would be I suggest, a significant step toward the desired theory of quantum gravity.

References

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[4] D. Shanahan,The de Broglie wave as an undulatory distortion induced in the moving particle by the failure of simultaneity, based on a presentation at the Sorbonne in May 2023 in honour of Louis de Broglie, to be published in Annales de la Fondation Louis de Broglie