Minkowski's actual and intended contributions to spacetime physics

Vesselin Petkov
Minkowski Institute
Montreal, Quebec, Canada
http://minkowskiinstitute.com
vpetkov@minkowskiinstitute.com

Hermann Mnikowski was born on June 22, 1864 and departed from this strange world on January 12, 1909 at the age of 44.

This year, when we mark the 160th anniversary of his birth, it is natural to recall again Minkowski's epoch-making contributions to spacetime physics. Given the depth of Minkowski's thinking and the far-reaching consequences of his results, it is also natural to wonder how fundamental (at least spacetime) physics might look like now if he had lived longer. In this sense, it is worth trying to imagine what Minkowski's intended[1] contributions might have been.

In the first part of the talk, I will discuss Minkowski's major contribution – the discovery of the spacetime structure of the world, i.e., the discovery of spacetime physics itself. In his Foreword to the second edition of the collection of papers *The Origin of Spacetime Physics* [1] Ashtekar nicely describes the application of the developed by Minkowski four-dimensional mathematical formalism of spacetime physics to electrodynamics:

> The new edition includes a Chapter based on Minkowkski's lecture to the Göttingen Scientific Society on December 21st, 1907, entitled *Fundamental Equations for the Electromagnetic Processes in Moving Bodies*. This is a much more detailed account of Minkowski's astonishingly deep understanding of how the fusion of space and time into a four-dimensional spacetime continuum leads to a reformulation of electrodynamics. In particular, this paper provides the tensorial formulation of Maxwell's equations and the action of the Lorentz group on the Maxwell field tensor and the source current. Because of its emphasis on four-dimensional geometry, this discussion of Maxwell's equations goes distinctly beyond Einstein's paper on *On the Electrodynamics of Moving Bodies*. Indeed, Minkowski's four-dimensional equations are exactly in the same form that we use today, more than a century later!

[1] I think the word "intended" is the appropriate one in view of Minkowski's program of geometrizing physics, which he intended to pursue.

After that I will summarize the evidence that Minkowski arrived independently of Einstein at the equivalence of the times of observers in relative motion and independently of Poincaré at the conclusion that the Lorentz transformations imply a four-dimensional space, but Einstein and Poincaré published first while Minkowski had been developing the full-blown four-dimensional formalism of spacetime physics; he did not publish his results earlier "because he wished first to work out the mathematical structure in all its splendour" (M. Born [2]).

In the second part of the talk I will discuss how applying his program of geometrizing physics[2] Minkowski (had he lived longer) might have arrived before Einstein (or independently of him) at the idea that gravitation is a manifestation of the non-Euclidean geometry of spacetiime.

References

[1] V. Petkov (ed.), *The Origin of Spacetime Physics*, 2nd ed. (Minkowski Institute Press, Montreal 2023).

[2] M. Born, *Einstein's Theory of Relativity* (Dover Publications, New York 1965), p. 33

[3] H. Minkowski, Space and Time, in [1, p. 148].

[2]After demonstrating that particles are a web of worldlines in a four-dimensional world (which Minkowski called *die Welt* and which we now call spacetime), Minkowski stated his program of regarding physics as spacetime geometry [3]:

> The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.