Enforcing the Unity of Space and Time using Quaternions

Gentlemen! And women! The radical change where "space by itself and time by itself will receded completely to become mere shadows" has yet to occur. Well-respected physicist continue to debate the problem of the arrow of time, when the discussion should be about the arrow of space-time.

Why are we still waiting?

The tools of tensor calculus are flexible in regards to dimensions. The Universe is not. If I were so radical as to erase all the Greek subscripts and superscripts from technical books and papers on physics, then I might be able to replace them all with a kind of four dimensional number known as a quaternion. A quaternion forces one to write time or time-related terms in the scalar part, and space or space-related terms in the 3-vector part of a quaternion. There is never a choice to omit. There are a number of immediate benefits to this Minkowski-friendly change of accounting systems. When Newton's second law is written out, there are two time derivatives that have no corresponding spatial derivative. The appearance of zeros or constants where operators could go shows an expression is classical. To generate the second law requires three zeros, so the law is as classical as can be. A relativistic expression on the other hand will have no space-time terms that are zero or constant. Take for example the difference between two arbitrarily close events in space-time and square it:

$$(dt,\ dx/c,\ dy/c,\ dt/c)^2 = (dt^2 - (dx^2 + dy^2 + dz^2)/c^2,\ 2\ dt\ dx/c,\ 2\ dt\ dy/c,\ 2dt\ dz/c)$$

Notice that the first term is identical to the Lorentz invariant interval for inertial observers, the symmetry at the heart of special relativity. There is now an opportunity for new physics by considering the a symmetry for the other three terms which I call space-times-time. The Schwarzschild solution of Einstein's general relativity theory for gravity almost leaves the space-times-time terms unchanged. I propose that the accidentally unchanged space-times-time is an an exact symmetry, the symmetry that gravity is about. Currently, gravity is the solution of ten nonlinear differential field equations. This proposal argues that gravity is a symmetry about all space-time algebra. In special relativity, one uses observers velocities to figure out how to conserve the interval. In this proposal, gravitational escape velocities do the same task to conserve space-times-time.

Radical ideas require much work. Such is the case for quantum mechanics which is usually written using a Hilbert vector space over a complex number field. I have made technical progress on this subject using series of quaternions to define precisely the inner product of two states: $< A|B >\equiv A^{*}B = \sum_{n}^{m} a_{n}^{*} b_{n}$. I have an iPython notebook which demonstrates different ways the quaternion representation is equivalent to the standard Hilbert space approach. Here again, when one writes out central equations in quantum mechanics such as the Schrödinger equation or the Klein-Gordon equation, one gets four equations instead of just one. We should expand Minkowski's vision of the profound union of time-like and space-like expressions into the quantum domain for fresh insights.